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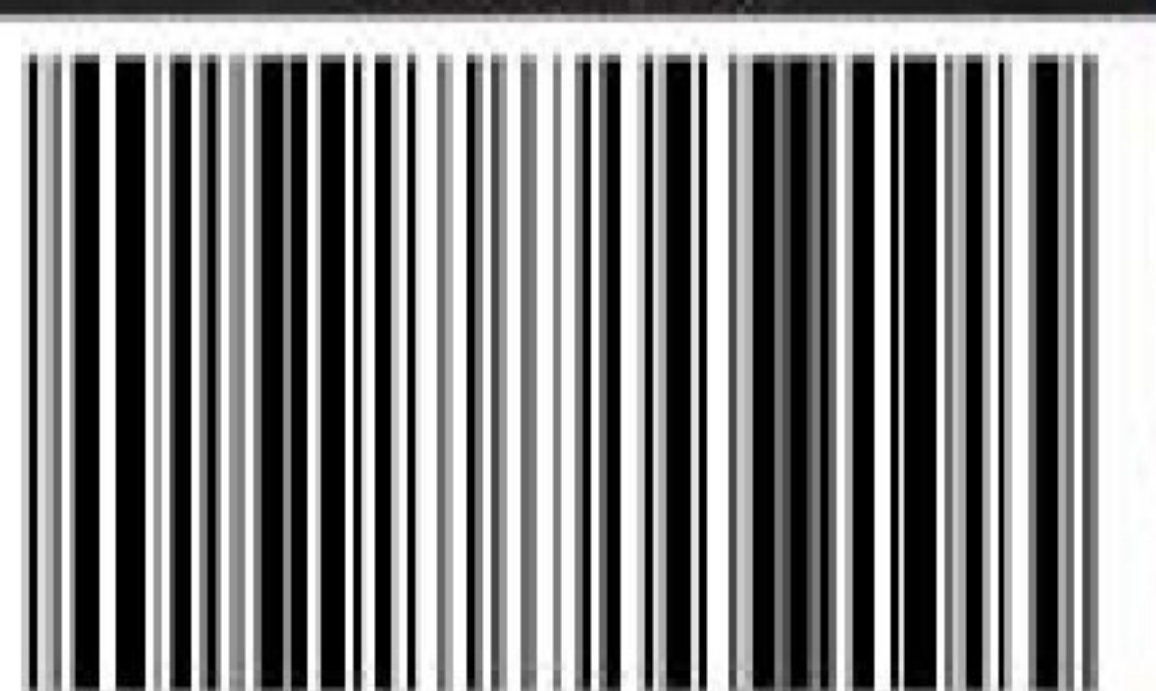
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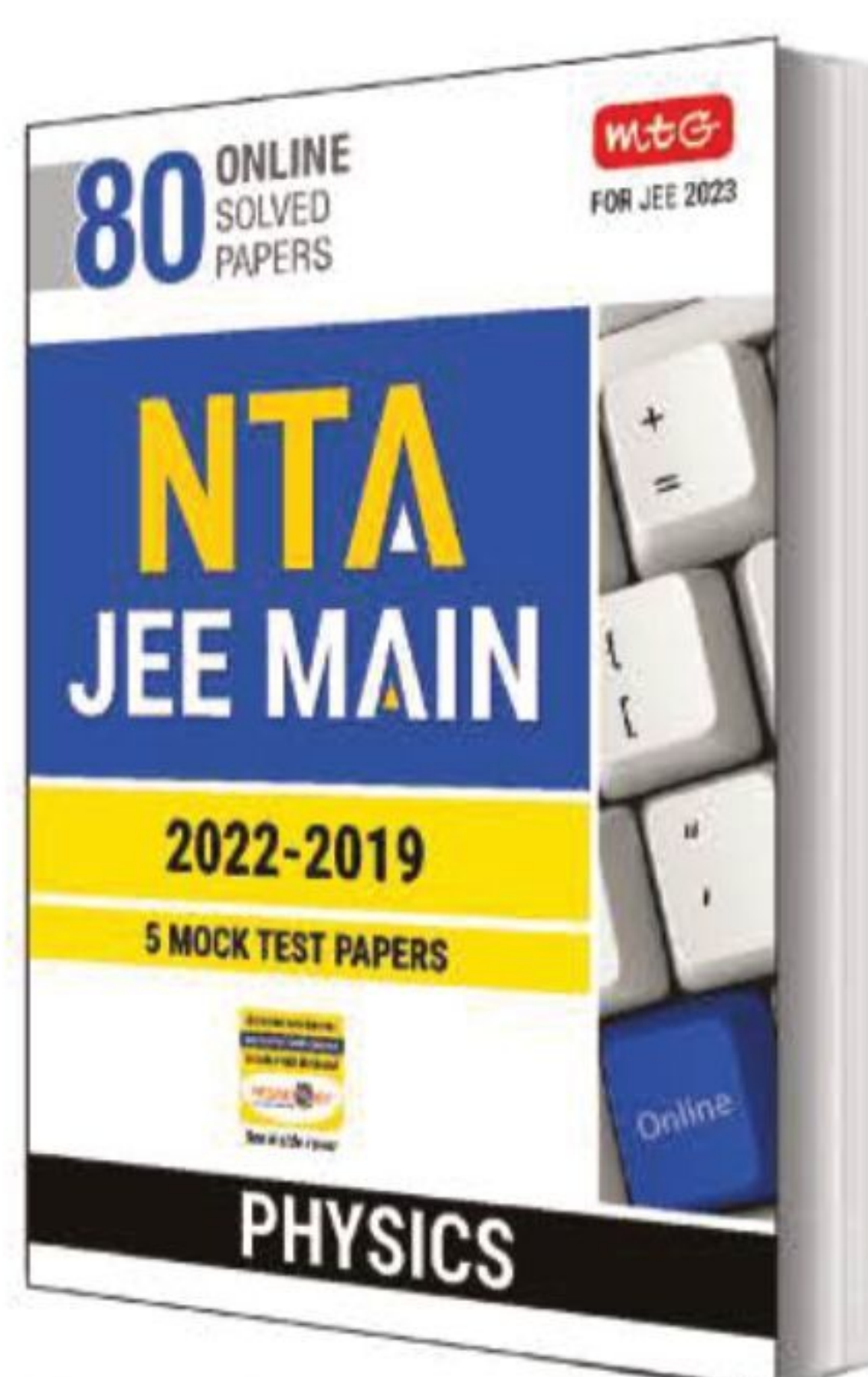
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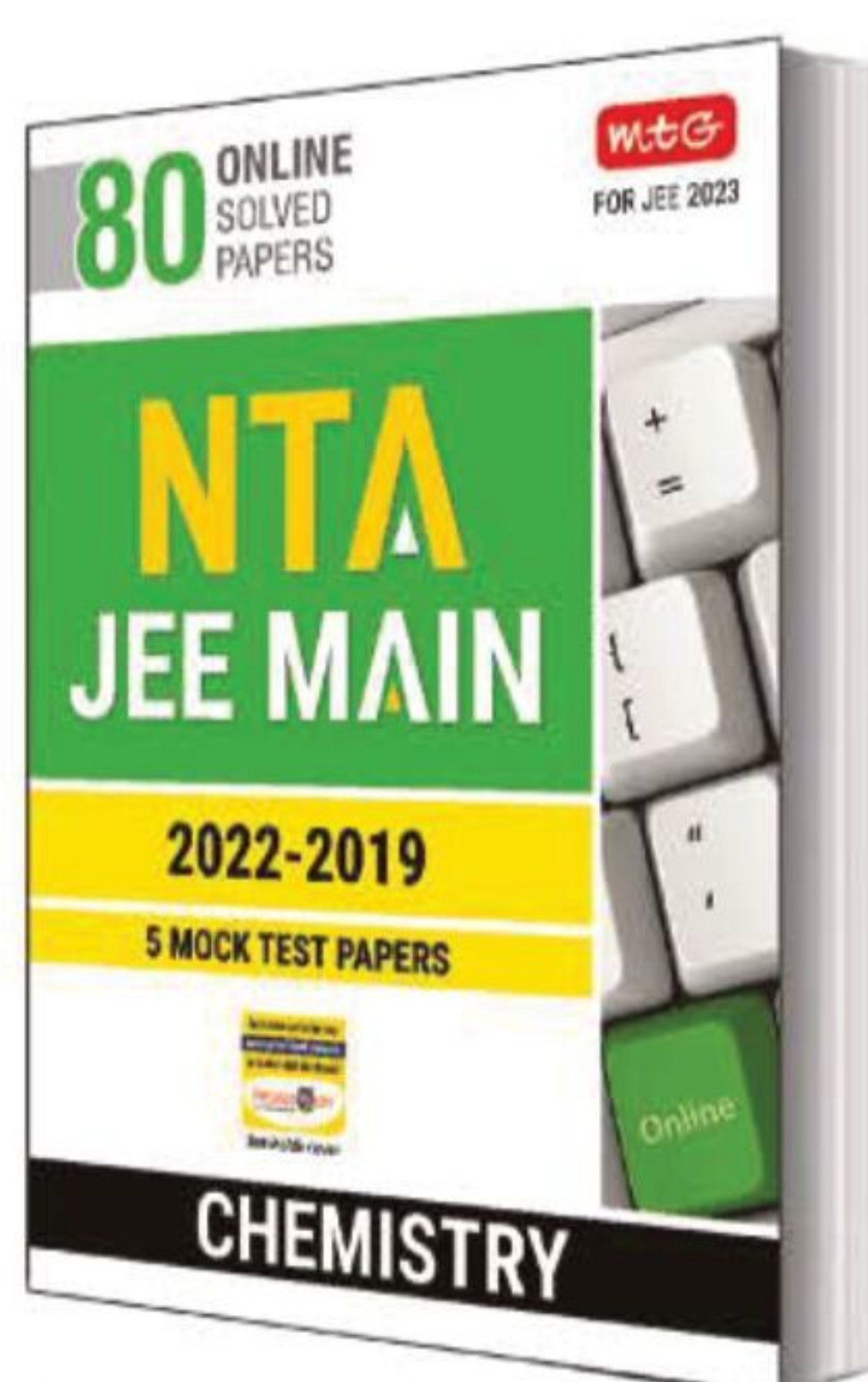
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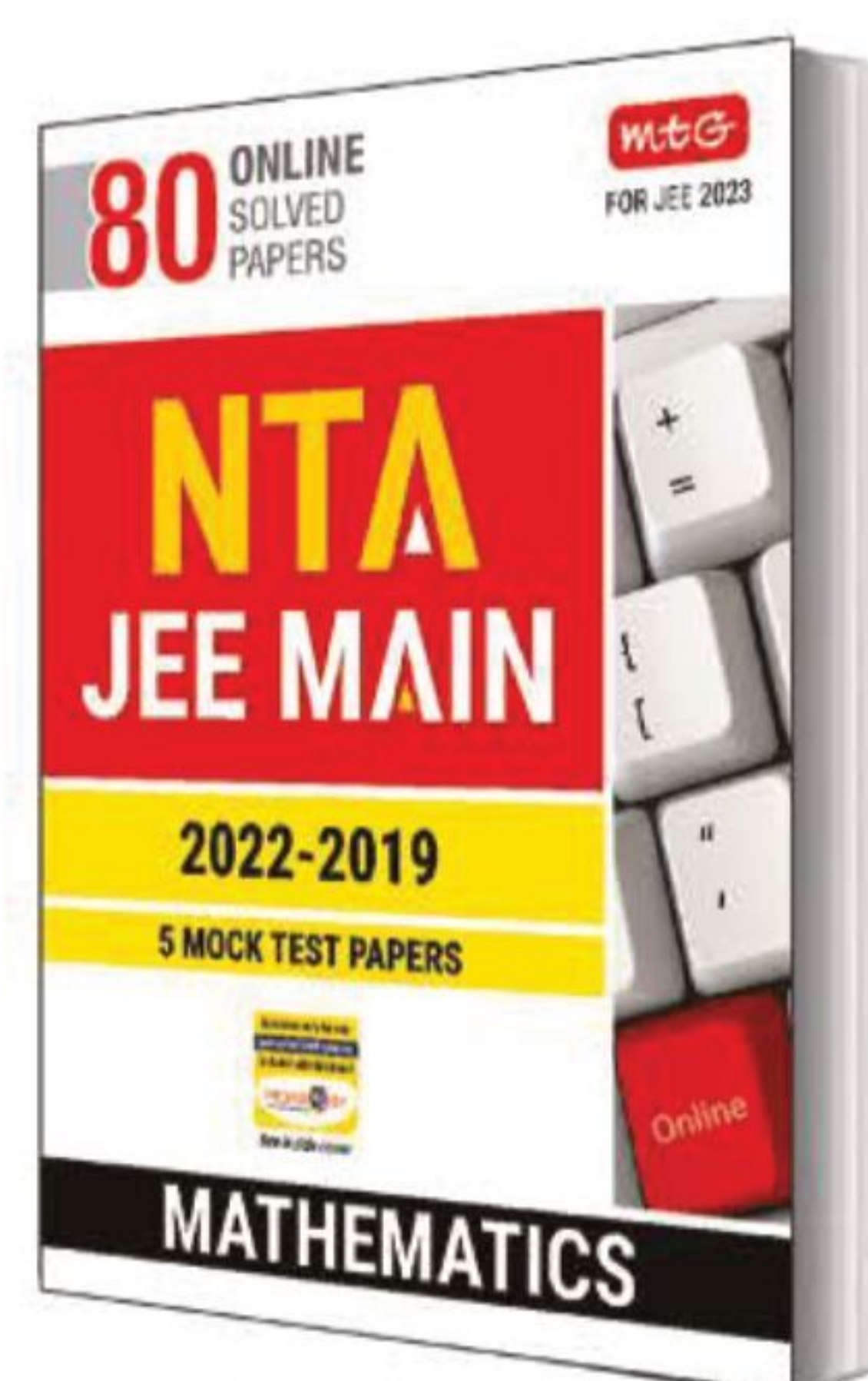
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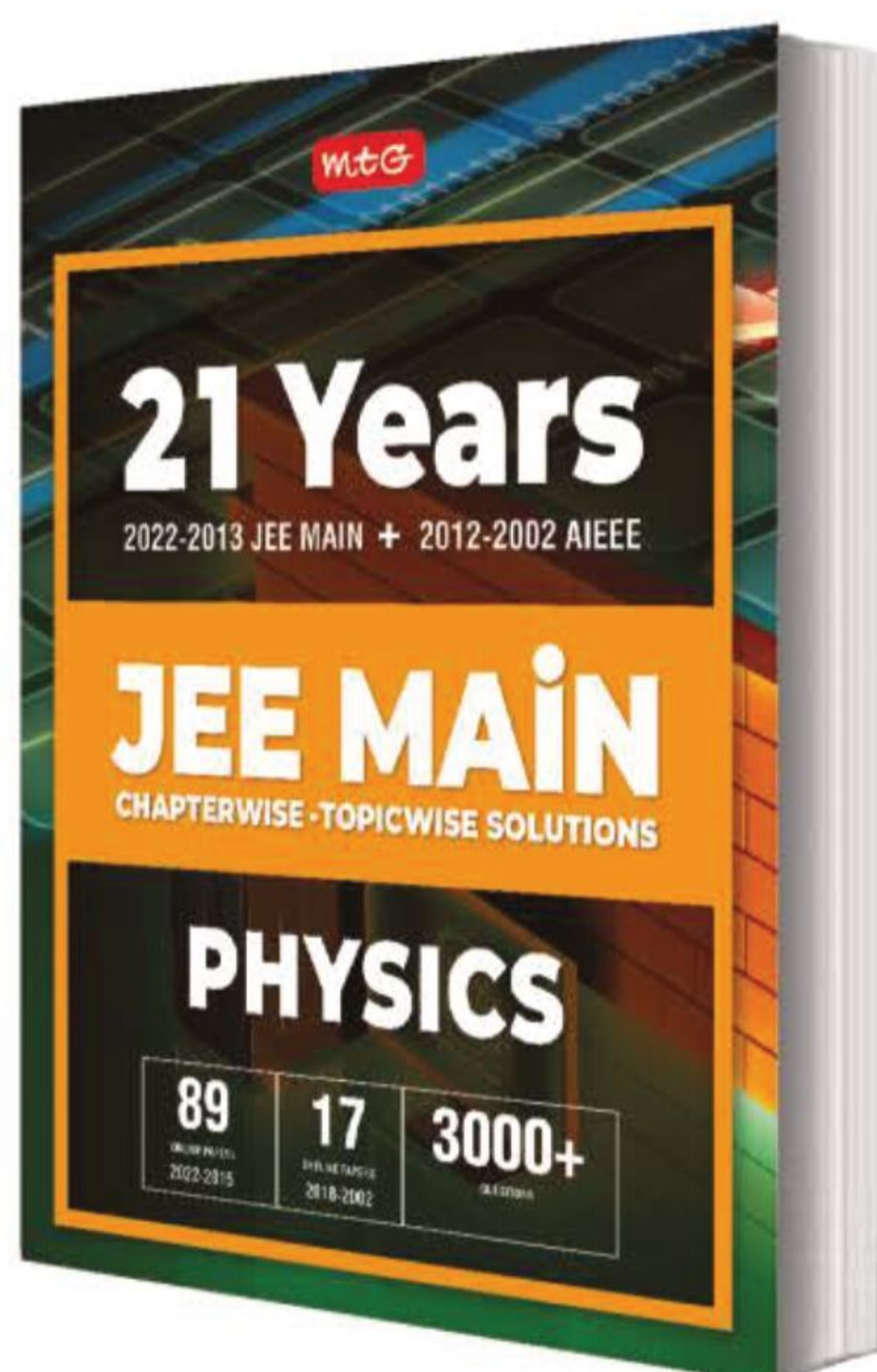


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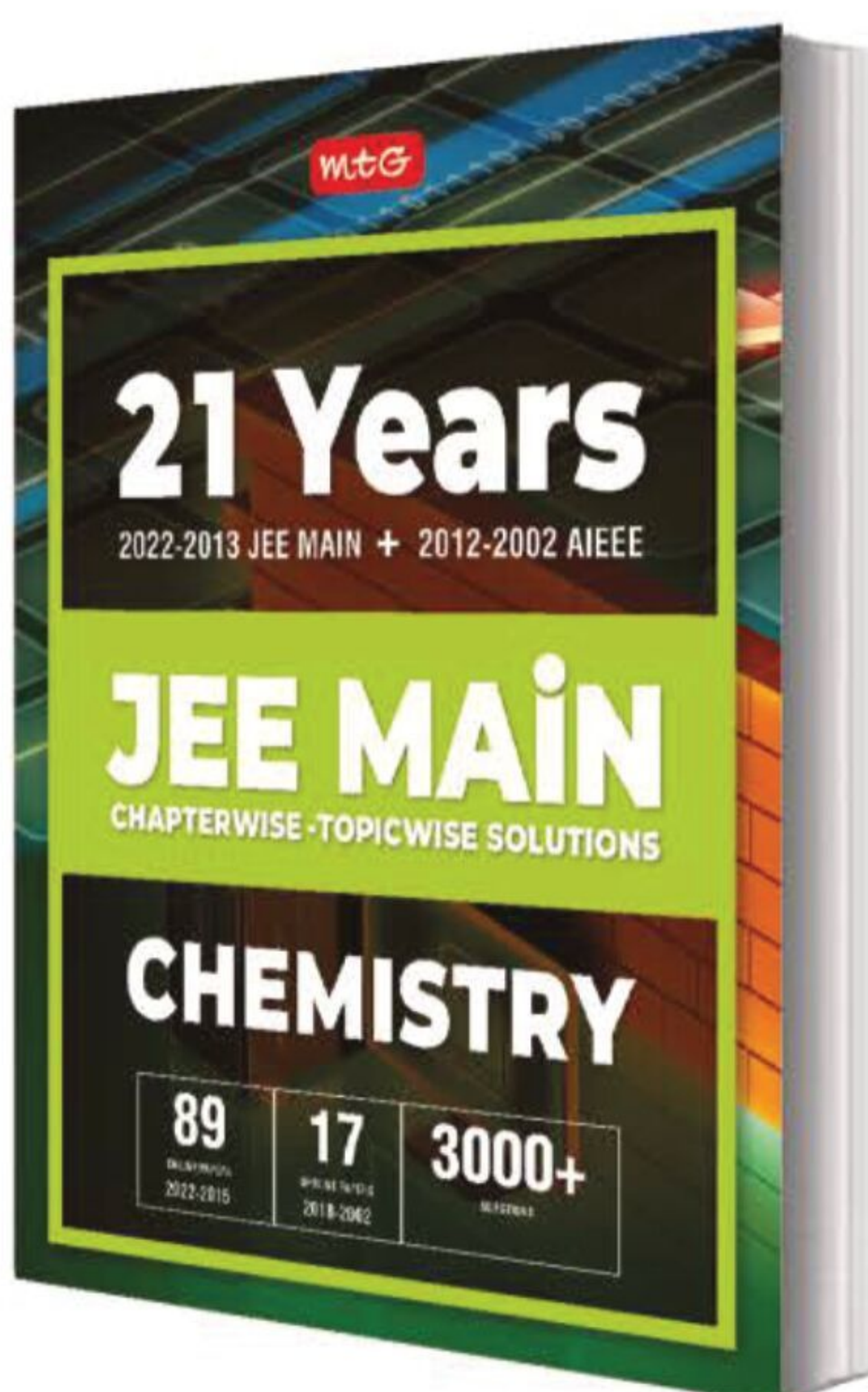


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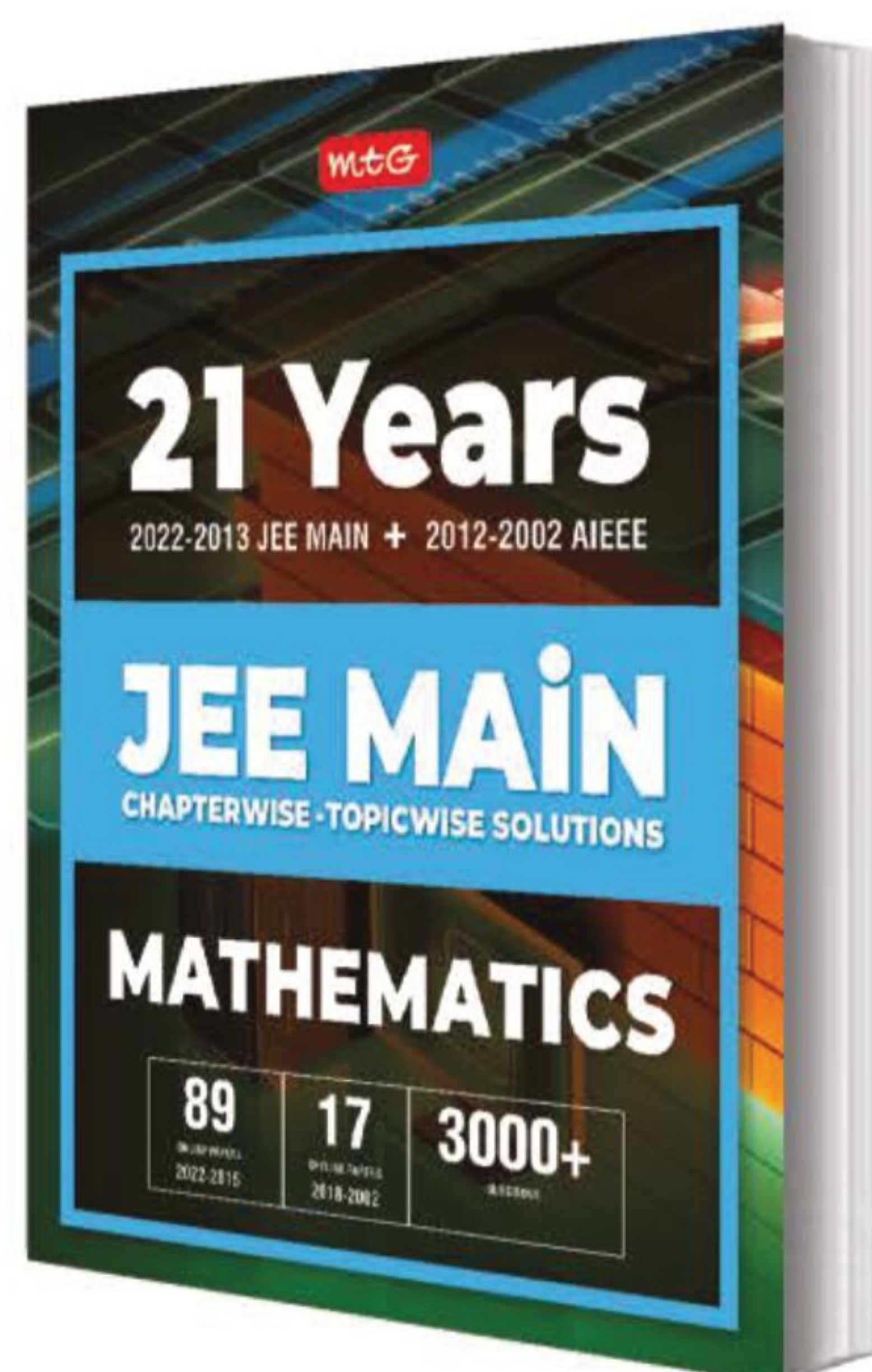
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Vol. XL No. 10 October 2022

Corporate Office:

Plot 99, Sector 44 Institutional Area,
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e-mail : info@mtg.in website : www.mtg.in

Regd. Office:

406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
Managing Editor : Mahabir Singh
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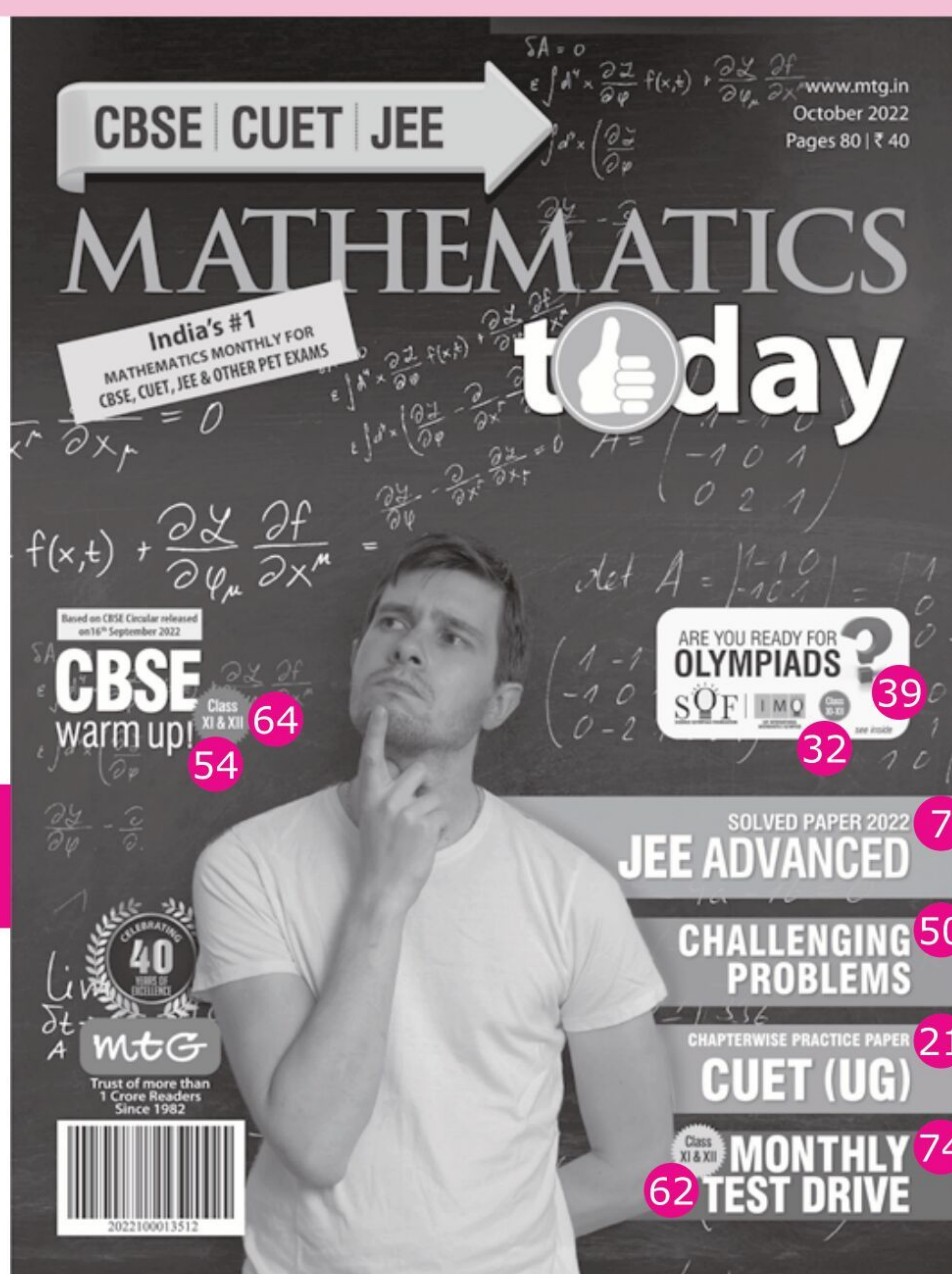
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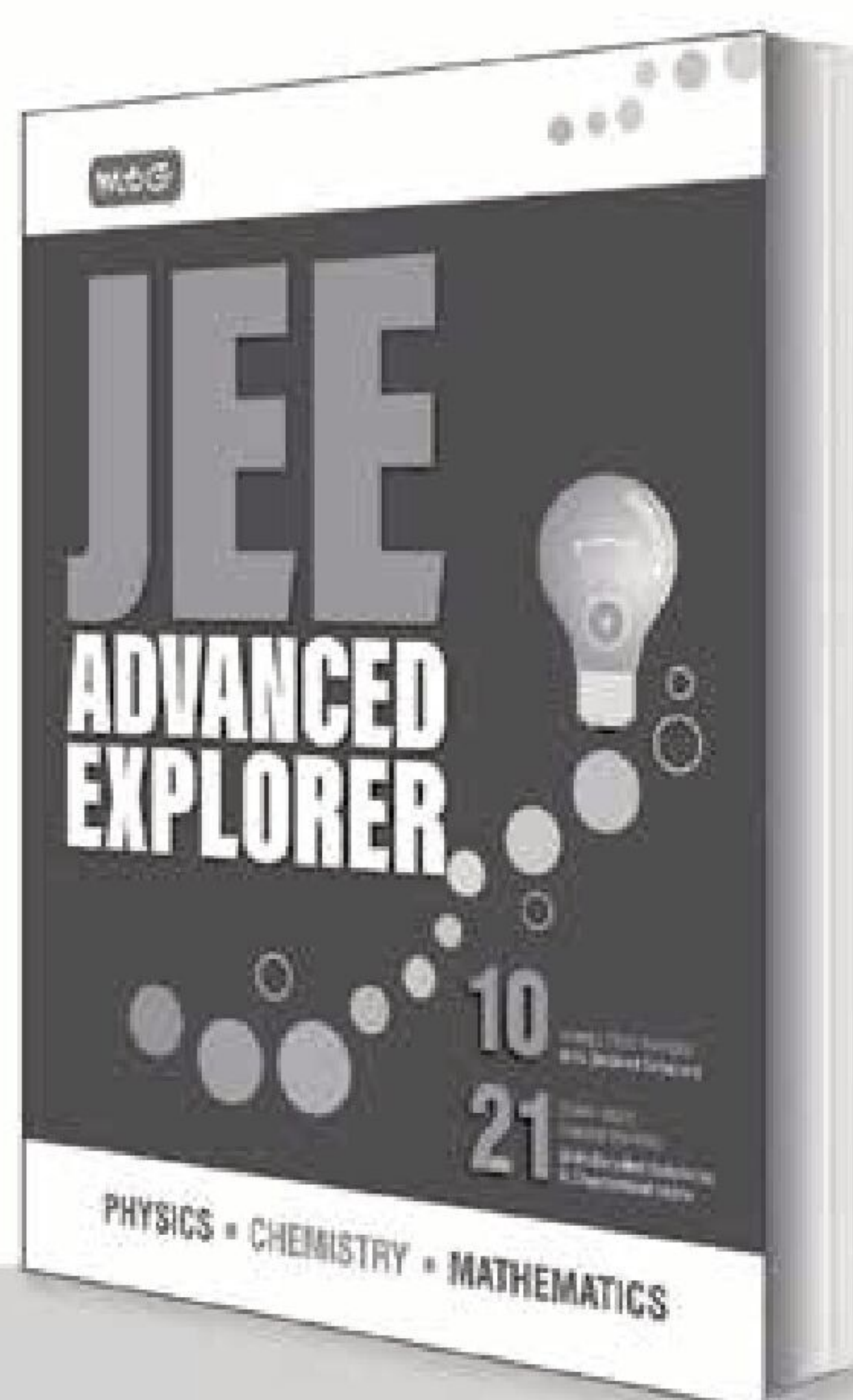
Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
Editor : Anil Ahlawat
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PAPER-1

SECTION 1 (MAXIMUM MARKS : 24)

- This section contains EIGHT (08) questions.
- The answer to each question is a NUMERICAL VALUE.
- If the numerical value has more than two decimal places, truncate/round off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 ONLY if the correct numerical value is entered;
Zero Marks : 0 In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\frac{3}{2} \cos^{-1} \sqrt{\frac{2}{2+\pi^2}} + \frac{1}{4} \sin^{-1} \frac{2\sqrt{2}\pi}{2+\pi^2} + \tan^{-1} \frac{\sqrt{2}}{\pi}$$

is _____.

2. Let α be a positive real number. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: (\alpha, \infty) \rightarrow \mathbb{R}$ be the functions defined by

$$f(x) = \sin\left(\frac{\pi x}{12}\right) \text{ and } g(x) = \frac{2 \log_e(\sqrt{x} - \sqrt{\alpha})}{\log_e(e^{\sqrt{x}} - e^{\sqrt{\alpha}})}.$$

Then the value of $\lim_{x \rightarrow \alpha^+} f(g(x))$ is _____.

3. In a study about a pandemic, data of 900 persons was collected. It was found that
190 persons had symptom of fever,
220 persons had symptom of cough,
220 persons had symptom of breathing problem,
330 persons had symptom of fever or cough or both,
350 persons had symptom of cough or breathing problem or both,
340 persons had symptom of fever or breathing problem or both,
30 persons had all the three symptoms (fever, cough and breathing problem).
If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is _____.

4. Let z be a complex number with non-zero imaginary part. If $\frac{2+3z+4z^2}{2-3z+4z^2}$ is a real number, then the value of $|z|^2$ is _____.

5. Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation $\bar{z} - z^2 = i(\bar{z} + z^2)$ is _____.

6. Let $l_1, l_2, \dots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1 , and let $w_1, w_2, \dots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2 , where $d_1 d_2 = 10$. For each $i = 1, 2, \dots, 100$, let R_i be a rectangle with length l_i , width w_i and area A_i . If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is _____.

7. The number of 4-digit integers in the closed interval $[2022, 4482]$ formed by using the digits 0, 2, 3, 4, 6, 7 is _____.

8. Let ABC be the triangle with $AB = 1$, $AC = 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB , AC and also touches internally the circumcircle of the triangle ABC , then the value of r is _____.

SECTION 2 (MAXIMUM MARKS : 24)

- This section contains SIX (06) questions.
- Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -2 In all other cases.

9. Consider the equation

$$\int_1^e \frac{(\log_e x)^{1/2}}{x(a - (\log_e x)^{3/2})^2} dx = 1, a \in (-\infty, 0) \cup (1, \infty).$$

Which of the following statements is/are true ?

- (a) No a satisfies the above equation
 (b) An integer a satisfies the above equation
 (c) An irrational number a satisfies the above equation
 (d) More than one a satisfy the above equation

10. Let $a_1, a_2, a_3, \dots$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, \dots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are true?

- (a) $T_{20} = 1604$ (b) $\sum_{k=1}^{20} T_k = 10510$
 (c) $T_{30} = 3454$ (d) $\sum_{k=1}^{30} T_k = 35610$

11. Let P_1 and P_2 be two planes given by

$$P_1 : 10x + 15y + 12z - 60 = 0,$$

$$P_2 : -2x + 5y + 4z - 20 = 0.$$

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2 ?

- (a) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$ (b) $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$
 (c) $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$ (d) $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

12. Let S be the reflection of a point Q with respect to the plane given by $\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$,

where t and p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i} + 15\hat{j} + 20\hat{k}$ and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the following is/are true ?

- (a) $3(\alpha + \beta) = -101$ (b) $3(\beta + \gamma) = -71$
 (c) $3(\gamma + \alpha) = -86$ (d) $3(\alpha + \beta + \gamma) = -121$

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point $P = (-2, 1)$ meet the parabola at P_1 and

P_2 . Let Q_1 and Q_2 be the points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2 . Then, which of the following is/are true?

- (a) $SQ_1 = 2$ (b) $Q_1Q_2 = \frac{3\sqrt{10}}{5}$
 (c) $PQ_1 = 3$ (d) $SQ_2 = 1$

14. Let $|M|$ denote the determinant of a square matrix M .

Let $g : \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be the function defined by

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$$

$$\text{where, } f(\theta) = \frac{1}{2} \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix} + \begin{vmatrix} \sin \pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos \frac{\pi}{2} & \log_e\left(\frac{\pi}{4}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan \pi \end{vmatrix}$$

Let $p(x)$ be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2) = 2 - \sqrt{2}$. Then, which of the following is/are true?

- (a) $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$ (b) $p\left(\frac{1+3\sqrt{2}}{4}\right) > 0$
 (c) $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$ (d) $p\left(\frac{5-\sqrt{2}}{4}\right) < 0$

SECTION 3 (MAXIMUM MARKS : 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Five entries (P), (Q), (R), (S) and (T).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :
Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

15. Consider the following lists

List-I		List-II	
(I)	$\left\{ x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3} \right] : \cos x + \sin x = 1 \right\}$	(P)	has two elements
(II)	$\left\{ x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18} \right] : \sqrt{3} \tan 3x = 1 \right\}$	(Q)	has three elements
(III)	$\left\{ x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5} \right] : 2 \cos(2x) = \sqrt{3} \right\}$	(R)	has four elements
(IV)	$\left\{ x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4} \right] : \sin x - \cos x = 1 \right\}$	(S)	has five elements
		(T)	has six elements

The correct option is :

- (a) (I) $\rightarrow$ (P); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (S)
 (b) (I) $\rightarrow$ (P); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (R)
 (c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (P); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (S)
 (d) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)

16. Two players, P_1 and P_2 , play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2 , respectively. If $x > y$, then P_1 scores 5 points and P_2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2 , respectively, after playing the i^{th} round.

List-I		List-II	
(I)	Probability of $(X_2 \geq Y_2)$ is	(P)	$\frac{3}{8}$
(II)	Probability of $(X_2 > Y_2)$ is	(Q)	$\frac{11}{16}$
(III)	Probability of $(X_3 = Y_3)$ is	(R)	$\frac{5}{16}$
(IV)	Probability of $(X_3 > Y_3)$ is	(S)	$\frac{355}{864}$
		(T)	$\frac{77}{432}$

The correct option is :

- (a) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (S)
 (b) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (T)
 (c) (I) $\rightarrow$ (P); (II) $\rightarrow$ (R); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (S)
 (d) (I) $\rightarrow$ (P); (II) $\rightarrow$ (R); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (T)

17. Let p, q, r be non-zero real numbers that are respectively, the 10^{th} , 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equation

$$\begin{aligned} x + y + z &= 1 \\ 10x + 100y + 1000z &= 0 \\ qrx + pry + pqz &= 0 \end{aligned}$$

List-I		List-II	
(I)	If $\frac{q}{r} = 10$, then the system of linear equation is	(P)	$x = 0, y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II)	If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q)	$x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III)	If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R)	infinitely many solutions
(IV)	If $\frac{p}{q} = 10$, then the system of linear equations has	(S)	no solution
		(T)	at least one solution

The correct option is :

- (a) (I) $\rightarrow$ (T); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (T)
 (b) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)
 (c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (R)
 (d) (I) $\rightarrow$ (T); (II) $\rightarrow$ (S); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (T)

18. Consider the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Let $H(\alpha, 0)$, $0 < \alpha < 2$, be a point. A straight line drawn through H parallel to the y -axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x -axis at a point G . Suppose the straight line joining F and the origin makes an angle ϕ with the positive x -axis.

List-I		List-II	
(I)	If $\phi = \frac{\pi}{4}$, then the area of the triangle FGH is	(P)	$\frac{(\sqrt{3}-1)^4}{8}$
(II)	If $\phi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q)	1
(III)	If $\phi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R)	$\frac{3}{4}$

(IV)	If $\phi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S)	$\frac{1}{2\sqrt{3}}$
		(T)	$\frac{3\sqrt{3}}{2}$

The correct option is :

- (a) (I) $\rightarrow$ (R); (II) $\rightarrow$ (S); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (P)
 (b) (I) $\rightarrow$ (R); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)
 (c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (T); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)
 (d) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (S); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (P)

PAPER-2

SECTION-I (MAXIMUM MARKS: 24)

- This section contains EIGHT (08) questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If ONLY the correct integer is entered;
 Zero Marks : 0 If the question is unanswered;
 Negative Marks : -1 In all other cases.

- Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha + \beta) = \frac{1}{3}$ and $\cos(\alpha - \beta) = \frac{2}{3}$, then the greatest integer less than or equal to $\left(\frac{\sin \alpha}{\cos \beta} + \frac{\cos \beta}{\sin \alpha} + \frac{\cos \alpha}{\sin \beta} + \frac{\sin \beta}{\cos \alpha} \right)^2$ is _____.
- If $y(x)$ is the solution of the differential equation $xy - (y^2 - 4y)dx = 0$ for $x > 0$, $y(1) = 2$, and the slope of the curve $y = y(x)$ is never zero, then the value of $10y(\sqrt{2})$ is _____.
- The greatest integer less than or equal to $\int_1^2 \log_2(x^3 + 1)dx + \int_1^{\log_2 9} (2^x - 1)^{1/3} dx$ is _____.
- The product of all positive real values of x satisfying the equation $x^{(16(\log_5 x)^3 - 68 \log_5 x)} = 5^{-16}$ is _____.
- If $\beta = \lim_{x \rightarrow 0} \frac{e^{x^3} - (1 - x^3)^{1/3} + ((1 - x^2)^{1/2} - 1) \sin x}{x \sin^2 x}$ then the value of 6β is _____.

- Let β be a real number. Consider the matrix

$$A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$$

If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix, then the value of 9β is _____.

- Consider the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$

with foci at S and S_1 , where S lies on the positive x -axis. Let P be a point on the hyperbola, in the first quadrant.

Let $\angle SPS_1 = \alpha$, with $\alpha < \frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola, intersects the straight line S_1P at P_1 . Let δ be the distance of P from the straight line SP_1 , and $\beta = S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9} \sin \frac{\alpha}{2}$ is _____.

- Consider the functions $f, g: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = x^2 + \frac{5}{12} \text{ and } g(x) = \begin{cases} 2\left(1 - \frac{4|x|}{3}\right), & |x| \leq \frac{3}{4}, \\ 0, & |x| > \frac{3}{4} \end{cases}$$

if α is the area of the region

$$\left\{ (x, y) \in \mathbb{R} \times \mathbb{R} : |x| \leq \frac{3}{4}, 0 \leq y \leq \min \{f(x), g(x)\} \right\},$$

then the value of 9α is _____.

SECTION-2 (MAXIMUM MARKS : 24)

- This section contains SIX (06) questions.
- Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer (s).

- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks: -2 In all other cases.

9. Let PQRS be a quadrilateral in a plane, where $QR = 1$, $\angle PQR = \angle QRS = 70^\circ$, $\angle PQS = 15^\circ$ and $\angle PRS = 40^\circ$. If $\angle RPS = \theta^\circ$, $PQ = \alpha$ and $PS = \beta$, then the interval(s) that contain(s) the value of $4\alpha\beta \sin\theta^\circ$ is/are

- (a) $(0, \sqrt{2})$ (b) $(1, 2)$
(c) $(\sqrt{2}, 3)$ (d) $(2\sqrt{2}, 3\sqrt{2})$

10. Let $\alpha = \sum_{k=1}^{\infty} \sin^{2k} \left(\frac{\pi}{6} \right)$

Let $g: [0, 1] \rightarrow R$ be the function defined by $g(x) = 2^{\alpha x} + 2^{\alpha(1-x)}$

Then, which of the following statements is/are true?

- (a) The minimum value of $g(x)$ is $2^{7/6}$
(b) The maximum value of $g(x)$ is $1 + 2^{1/3}$
(c) The function $g(x)$ attains its maximum at more one point
(d) The function $g(x)$ attains its minimum at more than one point

11. Let $\bar{z}$ denote the complex conjugate of a complex number z . If z is a non-zero complex number for which both real and imaginary parts of $(\bar{z})^2 + \frac{1}{z^2}$ are integers, then which of the following is/are possible value(s) of $|z|$?

- (a) $\left(\frac{43 + 3\sqrt{205}}{2} \right)^{1/4}$ (b) $\left(\frac{7 + \sqrt{33}}{4} \right)^{1/4}$
(c) $\left(\frac{9 + \sqrt{65}}{4} \right)^{1/4}$ (d) $\left(\frac{7 + \sqrt{13}}{6} \right)^{1/4}$

12. Let G be a circle of radius $R > 0$. Let $G_1, G_2, \dots, G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2, \dots, G_n$ touches the circle G externally. Also, for $i = 1, 2, \dots, n-1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are true?

- (a) If $n = 4$, then $(\sqrt{2} - 1)r < R$
(b) If $n = 5$, then $r < R$
(c) If $n = 8$, then $(\sqrt{2} - 1)r < R$
(d) If $n = 12$, then $\sqrt{2}(\sqrt{3} + 1)r > R$

13. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$$\begin{aligned}\vec{a} &= 3\hat{i} + \hat{j} - \hat{k}, \\ \vec{b} &= \hat{i} + b_2\hat{j} + b_3\hat{k}, \quad b_2, b_3 \in R, \\ \vec{c} &= c_1\hat{i} + c_2\hat{j} + c_3\hat{k}, \quad c_1, c_2, c_3 \in R\end{aligned}$$

be three vectors such that $b_2b_3 > 0$, $\vec{a} \cdot \vec{b} = 0$

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3 - c_1 \\ 1 - c_2 \\ -1 - c_3 \end{pmatrix}$$

Then, which of the following is/are true?

- (a) $\vec{a} \cdot \vec{c} = 0$ (b) $\vec{b} \cdot \vec{c} = 0$
(c) $|\vec{b}| > \sqrt{10}$ (d) $|\vec{c}| \leq \sqrt{11}$

14. For $x \in R$, let the function $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 12y = \cos \left(\frac{\pi}{12} x \right), y(0) = 0.$$

Then, which of the following statements is/are true?

- (a) $y(x)$ is an increasing function
(b) $y(x)$ is a decreasing function
(c) There exists a real number β such that the line $y = \beta$ intersects the curve the curve $y = y(x)$ at infinitely many points
(d) $y(x)$ is a periodic function

SECTION-3 (MAXIMUM MARKS : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Marks : -1 In all other cases.

15. Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen?

- (a) 21816 (b) 85536
(c) 12096 (d) 156816

16. If $M = \begin{pmatrix} 5/2 & 3/2 \\ -3/2 & -1/2 \end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

- (a) $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$ (b) $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$
(c) $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$ (d) $\begin{pmatrix} 3032 & 3032 \\ -3031 & -3030 \end{pmatrix}$

17. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,

Box-II contains 24 red, 9 blue and 15 green balls,

Box-III contains 1 blue, 12 green and 3 yellow balls,

Box-IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box-I; call this ball b . If b is red then a ball is chosen randomly from Box-II, if b is blue then a ball is chosen randomly from Box-III, and

if b is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one of the chosen balls is white' given that the event 'at least one of the chosen balls is green' has happened, is equal to

- (a) $\frac{15}{256}$ (b) $\frac{3}{16}$ (c) $\frac{5}{52}$ (d) $\frac{1}{8}$

18. For positive integer n , define

$$f(n) = n + \frac{16 + 5n - 3n^2}{4n + 3n^2} + \frac{32 + n - 3n^2}{8n + 3n^2} + \frac{48 - 3n - 3n^2}{12n + 3n^2} + \dots + \frac{25n - 7n^2}{7n^2}$$

Then, the value of $\lim_{n \rightarrow \infty} f(n)$ is equal to

- (a) $3 + \frac{4}{3} \log_e 7$ (b) $4 - \frac{3}{4} \log_e \left(\frac{7}{3}\right)$
(c) $4 - \frac{4}{3} \log_e \left(\frac{7}{3}\right)$ (d) $3 + \frac{3}{4} \log_e 7$

SOLUTIONS

PAPER-1

1. (2.36): $\cos^{-1} \sqrt{\frac{2}{2 + \pi^2}} = \tan^{-1} \frac{\pi}{\sqrt{2}}$

Also, $\sin^{-1} \left(\frac{2\sqrt{2}\pi}{2 + \pi^2} \right) = \sin^{-1} \left(\frac{2\pi/\sqrt{2}}{1 + (\pi/\sqrt{2})^2} \right)$
 $= \pi - 2 \tan^{-1} \frac{\pi}{\sqrt{2}} \quad \left(\text{As } \frac{\pi}{\sqrt{2}} > 1 \right)$

$\left[\text{Recall that } \sin^{-1} \frac{2x}{1+x^2} = \pi - 2 \tan^{-1} x, \text{ for } x \geq 1 \right]$

Now $\frac{3}{2} \cos^{-1} \sqrt{\frac{2}{2 + \pi^2}} + \frac{1}{4} \sin^{-1} \frac{2\sqrt{2}\pi}{2 + \pi^2} + \tan^{-1} \frac{\sqrt{2}}{\pi}$
 $= \frac{3}{2} \cdot \tan^{-1} \frac{\pi}{\sqrt{2}} + \frac{1}{4} \left(\pi - 2 \tan^{-1} \frac{\pi}{\sqrt{2}} \right) + \tan^{-1} \frac{\sqrt{2}}{\pi}$
 $= \tan^{-1} \frac{\pi}{\sqrt{2}} + \tan^{-1} \frac{\sqrt{2}}{\pi} + \frac{\pi}{4}$
 $= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} = 2.36 \quad (\text{Using } \pi = 3.141)$

[Rating : Easy]

2. (0.50): $\lim_{x \rightarrow \alpha^+} f(g(x)) = f \left(\lim_{x \rightarrow \alpha^+} g(x) \right)$

(As f is continuous on R)

Now $\lim_{x \rightarrow \alpha^+} g(x) = \lim_{x \rightarrow \alpha^+} \frac{2 \log_e (\sqrt{x} - \sqrt{\alpha})}{\log_e (e^{\sqrt{x}} - e^{\sqrt{\alpha}})}$
 $= \lim_{x \rightarrow \alpha^+} \frac{2 \left(\frac{1}{\sqrt{x} - \sqrt{\alpha}} \right) \frac{1}{2\sqrt{x}}}{\frac{1}{e^{\sqrt{x}} - e^{\sqrt{\alpha}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}$
 $= \frac{2}{e^{\sqrt{\alpha}}} \lim_{x \rightarrow \alpha^+} \frac{e^{\sqrt{x}} - e^{\sqrt{\alpha}}}{\sqrt{x} - \sqrt{\alpha}} = \frac{2}{e^{\sqrt{\alpha}}} \cdot \frac{e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}} = 2$

(By repeatedly applying L'Hospital's Rule)

So, $\lim_{x \rightarrow \alpha^+} f(g(x)) = \lim_{x \rightarrow \alpha^+} f(2)$

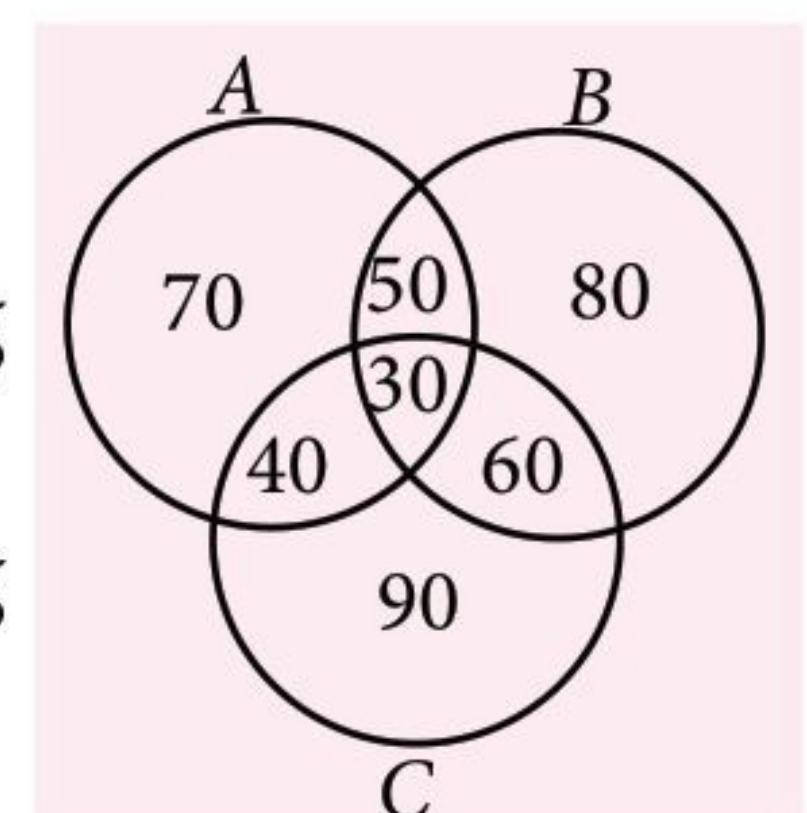
$\therefore$ Required limit $= f(2) = \sin \frac{\pi}{6} = \frac{1}{2} = 0.50$

[Rating : Medium]

3. (0.80): Let A denote the persons having symptom of fever.

B denote the persons having symptom of cough.

C denote the persons having symptom of breathing problem.



Given $n(A) = 190$, $n(B) = 220$, $n(C) = 220$,
 $n(A \cup B) = 330$, $n(B \cup C) = 350$, $n(C \cup A) = 340$,
 $n(A \cap B \cap C) = 30$ and $n(S) = 900$

We have $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
 $= 190 + 220 - 330 = 80$

Similarly, $n(B \cap C) = 90$ and $n(C \cap A) = 70$

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 190 + 220 + 220 - (80 + 90 + 70) + 30$

$\therefore = 420$

So, $n((A \cup B \cup C)') = n(S) - n(A \cup B \cup C)$
 $= 900 - 420 = 480$

Thus, required probability $= \frac{70 + 80 + 90 + 480}{900}$

$$= \frac{720}{900} = \frac{4}{5} = 0.80$$

[Rating : Medium]

$$4. (0.50) : \frac{2 + 3z + 4z^2}{2 - 3z + 4z^2} = 1 + \frac{6z}{2 - 3z + 4z^2}$$

$$= 1 + 6 \frac{z}{2 - 3z + 4z^2}$$

For the expression to be real, $\frac{z}{2 - 3z + 4z^2}$ must be real.

$$\text{Hence, } \frac{z}{2 - 3z + 4z^2} = \frac{\bar{z}}{2 - 3\bar{z} + 4\bar{z}^2}$$

$$\Rightarrow 2z - 3z\bar{z} + 4z\bar{z}^2 = 2\bar{z} - 3z\bar{z} + 4z^2\bar{z},$$

which becomes $2(z - \bar{z}) - 4z\bar{z}(z - \bar{z}) = 0$

$$\Rightarrow 2(z - \bar{z})(1 - 2z\bar{z}) = 0$$

As $z \neq \bar{z}$ (Given) $\therefore$ We have $1 - 2z\bar{z} = 0$

$$\Rightarrow z\bar{z} = \frac{1}{2} \Rightarrow |z|^2 = \frac{1}{2} = 0.50 \quad [\text{Rating : Medium}]$$

$$5. (4) : \bar{z} - z^2 = i(\bar{z} + z^2) \Rightarrow \bar{z}(1 - i) = (1 + i)z^2$$

$$\Rightarrow z^2 = \frac{1 - i}{1 + i} \bar{z} \Rightarrow z^2 = -i\bar{z}$$

Let $z = x + iy$

$$\therefore (x^2 - y^2) + i(2xy) = -i(x - iy)$$

$$\text{So, } x^2 - y^2 + y = 0 \quad \dots(i)$$

$$\text{and } x(2y + 1) = 0 \quad \dots(ii)$$

$x = 0$, gives $y = 0, 1$

$$\text{and } y = -\frac{1}{2} \text{ gives } x = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

So, we get 4 solutions of ordered pair (x, y) .

[Rating : Challenging]

$$6. (18900) : A_{51} - A_{50} = (l_1 + 50d_1)(w_1 + 50d_2) - (l_1 + 49d_1)(w_1 + 49d_2)$$

$$= l_1d_2 + w_1d_1 + (50^2 - 49^2)d_1d_2 = 1000$$

$$\therefore l_1d_2 + w_1d_1 = 10 \quad (\because d_1d_2 = 10)$$

$$\text{Now, } A_{100} - A_{90} = (l_1 + 99d_1)(w_1 + 99d_2) - (l_1 + 89d_1)(w_1 + 89d_2)$$

$$= 10(l_1d_2 + w_1d_1) + (99^2 - 89^2)d_1d_2$$

$$= (10)(10) + (1880)(10) = 18900 \quad [\text{Rating : Easy}]$$

$$7. (569) : \underline{a} \quad \underline{b} \quad \underline{c} \quad \underline{d}$$

Case-I $a = 2$, $b = 0$, $c = 2$, d can take values 2, 3, 4, 6, 7
 $\rightarrow$ so 5 numbers

$a = 2$, $b = 0$, c can take values 3, 4, 6, 7, d has 6 choices
 $\rightarrow$ so 24 numbers

Case-II $a = 2$, b can take values 2, 3, 4, 6, 7, c and d have 6 choices each
 $\rightarrow$ so $5 \cdot 6 \cdot 6 = 180$ numbers

Case-III $a = 3$, b, c, d each has 6 choices $\rightarrow$ so $1 \cdot 6^3 = 216$ numbers

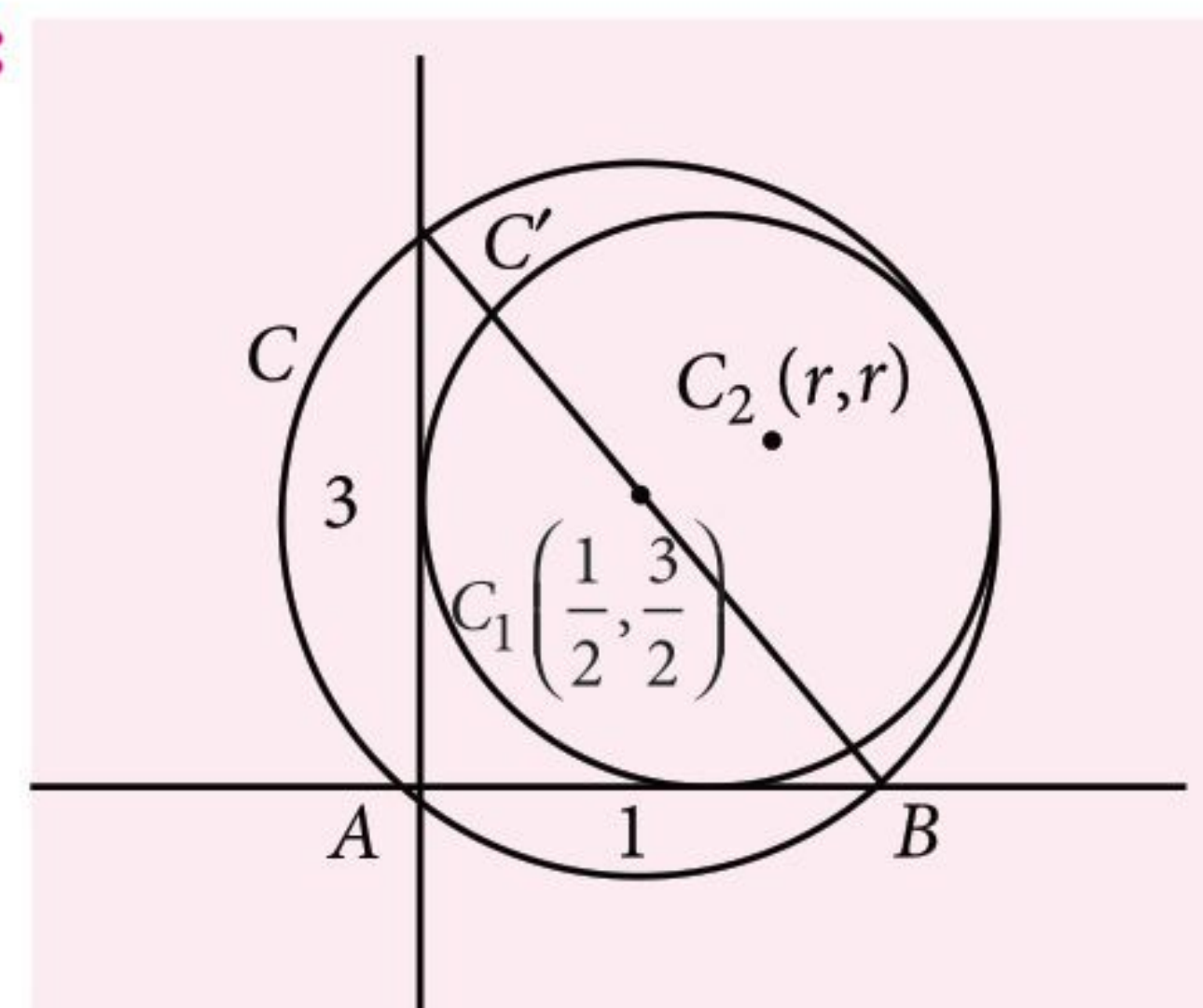
Case-IV $a = 4$, $b = 0, 2, 3, 4$, c and d have 6 choices each
 $\rightarrow$ so $4 \cdot 6^2 = 144$ numbers

The total number of 4 digit integers

$$= 5 + 24 + 180 + 216 + 144 = 569$$

[Rating : Challenging]

$$8. (0.84) :$$



Let $C_2 \equiv (r, r)$ C' : the required circle

$C_1 \equiv \left(\frac{1}{2}, \frac{3}{2}\right)$ C : the circumcircle of ΔABC .

Now, $C_1C_2 = \left|r - \frac{\sqrt{10}}{2}\right|$, for internal touching of C and C'

$$\Rightarrow \left(r - \frac{1}{2}\right)^2 + \left(r - \frac{3}{2}\right)^2 = \left(r - \frac{\sqrt{10}}{2}\right)^2,$$

which gives $r^2 - 4r + \sqrt{10}r = 0 \Rightarrow r(r - 4 + \sqrt{10}) = 0$

So $r = 4 - \sqrt{10}$ ($r > 0$)

[Rating : Challenging]

$$9. (c, d) : \int_1^e \frac{\sqrt{\log_e x}}{x(a - (\log_e x)^{3/2})^2} dx = 1$$

$$\text{Let } a - (\log_e x)^{3/2} = u, \Rightarrow -\frac{3}{2}(\log_e x)^{1/2} \cdot \frac{1}{x} dx = du$$

Thus, given equation becomes

$$-\frac{2}{3} \int_a^{a-1} \frac{du}{u^2} = 1 \Rightarrow -\frac{2}{3}(-1) \left[\frac{1}{u} \right]_a^{a-1} = 1$$

$$\Rightarrow \frac{2}{3a(a-1)} = 1 \Rightarrow 3a^2 - 3a - 2 = 0$$

$$\Rightarrow a = \frac{3 \pm \sqrt{33}}{6}$$

[Rating : Medium]

10. (b, c) : Given $T_{n+1} - T_n = a_n$ $n \geq 1$

Put $n = 1, 2$, in succession and adding, we get

$$T_2 - T_1 = a_1; T_3 - T_2 = a_2; T_n - T_{n-1} = a_{n-1}$$

So, $T_n = T_1 + a_1 + a_2 + \dots + a_{n-1} = 3 + 7 + 15 + \dots$ to $(n-1)$ term $= 3 + (n-1)(4n-1)$

$$= 4n^2 - 5n + 4, n \geq 1$$

$$\text{Also, } \sum_{k=1}^n T_k = \sum_{k=1}^n (4k^2 - 5k + 4)$$

$$= \frac{4n(n+1)(2n+1)}{6} - 5 \frac{n(n+1)}{2} + 4n$$

$$= \frac{n(n+1)}{6} \{4(2n+1) - 15\} + 4n = \frac{n(n+1)}{6} (8n-11) + 4n$$

$$\text{Thus, } T_{20} = 4 \times 20^2 - 5 \times 20 + 4 = 1504$$

$$\text{and } T_{30} = 4 \times 30^2 - 5 \times 30 + 4 = 3454$$

Also we can compute

$$\sum_{k=1}^{20} T_k = 10510; \sum_{k=1}^{30} T_k = 35615$$

[Rating : Medium]

11. (a, b, d) : The planes are $P_1 : 10x + 15y + 12z - 60 = 0$
 $P_2 : -2x + 5y + 4z - 20 = 0$

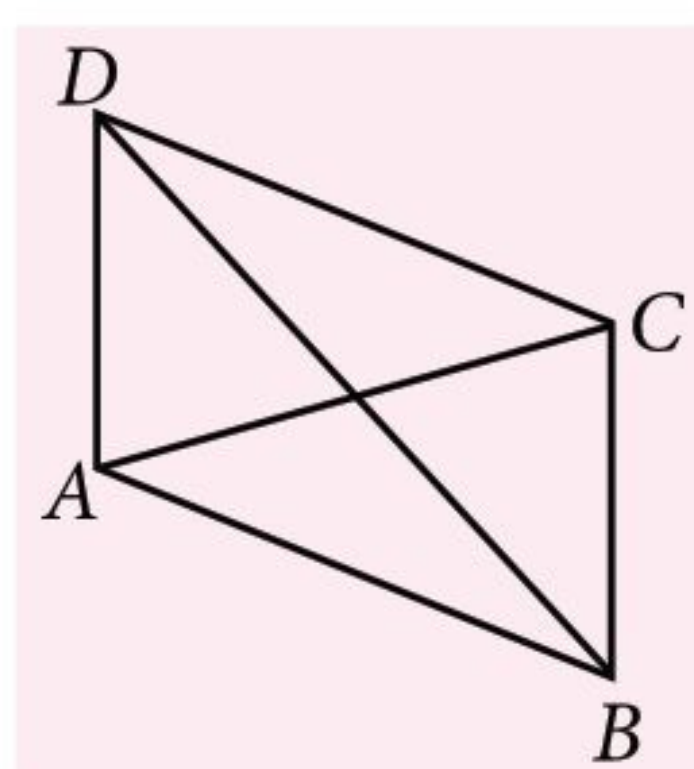
The line of intersection of planes P_1 and P_2 is

$$\frac{x}{15 \times 4 - 12 \times 5} = \frac{y}{-12 \times 2 - 10 \times 4} = \frac{z}{10 \times 5 + 2 \times 15}$$

$$\Rightarrow \frac{x}{0} = \frac{y}{-64} = \frac{z}{80} \Rightarrow \frac{x}{0} = \frac{y}{-4} = \frac{z}{5}$$

Let P_1 be DAB and P_2 be the DCB .

Any skew line with the line of intersection of plane P_1 and P_2 can be an edge of tetrahedron.



Also, any line intersecting the line of intersection and lying in plane P_1 or P_2 can be edge of the tetrahedron.

So, options (a), (b) and (d) are correct.

[Rating : Challenging]

12. (a, b, c) : The plane $\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$
 $= \hat{k} + t(-\hat{i} + \hat{j}) + p(-\hat{i} + \hat{k})$

The plane passes through $(0, 0, 1)$ and contains vectors $\langle -1, 1, 0 \rangle$ and $\langle -1, 0, 1 \rangle$.

The plane is $x + y + z = 1$

The reflection of $Q(10, 15, 20)$ in the plane, $S(\alpha, \beta, \gamma)$ is given by

$$\frac{\alpha-10}{1} = \frac{\beta-15}{1} = \frac{\gamma-20}{1} = -2 \left(\frac{10+15+20-1}{1+1+1} \right)$$

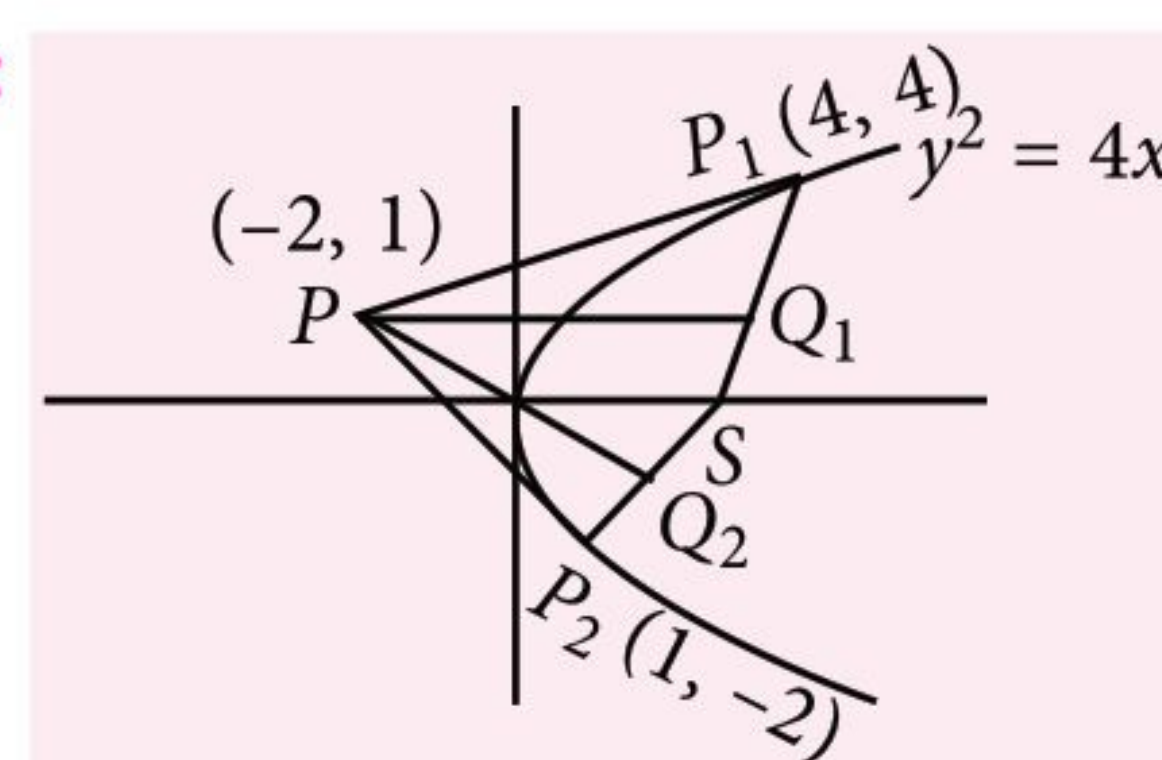
$$= \frac{-88}{3}$$

$$\text{So, } (\alpha, \beta, \gamma) \equiv \left(-\frac{58}{3}, -\frac{43}{3}, -\frac{28}{3} \right)$$

Thus, options (a), (b) and (c) are correct

[Rating : Easy]

13. (b, c, d) :



Equation of tangent at point 't' of the parabola is

$$x - ty = t^2 - 2t^2$$

$$\Rightarrow x - ty + t^2 = 0$$

It passes through $(-2, 1) \Rightarrow -2 - t + t^2$

$$\Rightarrow t^2 - t - 2 = 0 \Rightarrow (t-2)(t+1) = 0$$

$$\Rightarrow t = 2, -1$$

The points are $(t^2, 2t)$ i.e., $(4, 4)$ and $(1, -2)$

So, $P_1(4, 4)$ and $P_2(1, -2)$

Also, $S \equiv (1, 0)$

The equation of SP_1 is $4x - 3y - 4 = 0$

The equation of SP_2 is $x - 1 = 0$

$$Q_1 \text{ is given by } \frac{x+2}{4} = \frac{y-1}{-3} = \frac{-8-3-4}{25} = \frac{3}{5}$$

$$\text{i.e., } x = \frac{2}{5}, y = -\frac{4}{5} \text{ i.e., } Q_1 \equiv \left(\frac{2}{5}, -\frac{4}{5} \right)$$

$$Q_2 \text{ is given by } \frac{x+2}{1} = \frac{y-1}{0} = \frac{-2-1}{1}$$

$$\text{i.e., } x = 1, y = 1 \text{ i.e., } Q_2 \equiv (1, 1)$$

$$SQ_1 = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{3^2 + 4^2}{5^2}} = 1$$

$$SQ_2 = \sqrt{0^2 + 1^2} = 1$$

$$Q_1Q_2 = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{9}{5}\right)^2} = \left(\frac{3}{5}\right)\sqrt{1+3^2} = \frac{3\sqrt{10}}{5}$$

$$PQ_1 = \sqrt{\left(-2 - \frac{2}{5}\right)^2 + \left(1 + \frac{4}{5}\right)^2} = \sqrt{\frac{12^2 + 9^2}{5^2}} = 3$$

[Rating : Medium]

$$14. (a, c) : \frac{1}{2} \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$$

$$= \frac{1}{2}(2(1 + \sin^2 \theta)) = 1 + \sin^2 \theta$$

$$\therefore f(\theta) = \frac{1}{2} \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} \sin \pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos \frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan \pi \end{vmatrix}$$

$$= 1 + \sin^2 \theta + 0 \quad (\because \text{the second determinant is skew symmetric of odd order})$$

$$= 1 + \sin^2 \theta$$

Now, $g(\theta) = |\sin \theta| + |\cos \theta|$. So, $g(\theta) \in [1, \sqrt{2}]$

$$p(x) = \lambda(x-1)(x-\sqrt{2})$$

Given, $p(2) = 2 - \sqrt{2} = \lambda(1)(2 - \sqrt{2}) \therefore \lambda = 1$

$$\text{So, } P(x) = (x - \sqrt{2})(x - 1)$$

Thus, options (a) and (c) are correct.

[Rating : Challenging]

15. (b) : (I) $\cos x + \sin x = 1 \quad \forall x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}. \text{ So, } x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi, x = 2n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$$

Thus, $x = 0, \frac{\pi}{2}$ are 2 solutions in the given range.

(II) $\sqrt{3} \tan 3x = 1 \quad \forall x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right]$

$$\Rightarrow \tan 3x = \frac{1}{\sqrt{3}} = \tan \frac{\pi}{6}$$

$$\Rightarrow 3x = n\pi + \frac{\pi}{6} \Rightarrow x = (6n+1)\frac{\pi}{18}, n \in \mathbb{Z}$$

Thus, $x = -\frac{5\pi}{18}, \frac{\pi}{18}$ are 2 solutions in the given range.

(III) $2\cos 2x = \sqrt{3} \quad \forall x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right]$

$$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z} \Rightarrow x = n\pi \pm \frac{\pi}{12}$$

So, $x = \pm \frac{\pi}{12}, \pi \pm \frac{\pi}{12}, -\pi \pm \frac{\pi}{12}$ are 6 solutions in the given range.

(IV) $\sin x - \cos x = 1 \quad \forall x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right]$

$$\Rightarrow \cos x - \sin x = -1$$

$$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos \frac{3\pi}{4}$$

$$\Rightarrow x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}, n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{2}, 2n\pi - \pi, n \in \mathbb{Z}$$

So, $x = \frac{\pi}{2}, -\frac{3\pi}{2}, \pi, -\pi$ are 4 solutions in the given range.

[Rating : Easy]

16. (a) : A win gets 5 points, a loss gets 0 points and a draw gets 2 points.

$$P(D) = \frac{1}{6}, P(W) = \frac{5}{12} \text{ and } P(L) = \frac{5}{12}$$

$$P(X_2 > Y_2) = P(10, 0) + P(7, 2)$$

$$= \frac{5}{12} \cdot \frac{5}{12} + \frac{5}{12} \cdot \frac{1}{6} \cdot 2 = \frac{45}{144} = \frac{5}{16}$$

$$P(X_2 = Y_2) = P(5, 5) + P(4, 4) = \frac{5}{12} \cdot \frac{5}{12} \cdot 2 + \frac{1}{6} \cdot \frac{1}{6} = \frac{27}{72} = \frac{3}{8}$$

$$P(X_2 \geq Y_2) = \frac{5}{16} + \frac{3}{8} = \frac{11}{16}$$

$$P(X_3 = Y_3) = P(6, 6) + P(7, 7)$$

$$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} + \frac{5}{12} \cdot \frac{1}{6} \cdot \frac{5}{12} \cdot 6 = \frac{1}{216} + \frac{25}{144} = \frac{77}{432}$$

$$P(X_3 > Y_3) = \frac{1}{2} \left(1 - \frac{77}{432}\right) = \frac{1}{2} \cdot \frac{355}{432} = \frac{355}{864}$$

[Rating : Challenging]

17. (b) : The given equation in matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 10 & 10^2 & 10^3 \\ \frac{1}{p} & \frac{1}{q} & \frac{1}{r} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 10^2 & 10^3 \\ \frac{1}{p} & \frac{1}{q} & \frac{1}{r} \end{vmatrix} \quad D_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 10^2 & 10^3 \\ 0 & \frac{1}{q} & \frac{1}{r} \end{vmatrix}$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 0 & 10^3 \\ \frac{1}{p} & 0 & \frac{1}{r} \end{vmatrix} \quad D_z = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 10^2 & 0 \\ \frac{1}{p} & \frac{1}{q} & 0 \end{vmatrix}$$

(I) If $\frac{q}{r} = 10$, then $D_x = 0$, $D_y \neq 0$, $D_z \neq 0$ and

$D \neq 0$, so we have infinitely many solutions.

One can find the representative solutions.

(II) If $\frac{p}{r} \neq 100$, then $D_y \neq 0$

Hence, the system has no solution.

(III) If $\frac{p}{q} \neq 100$, then $D_z = 0$

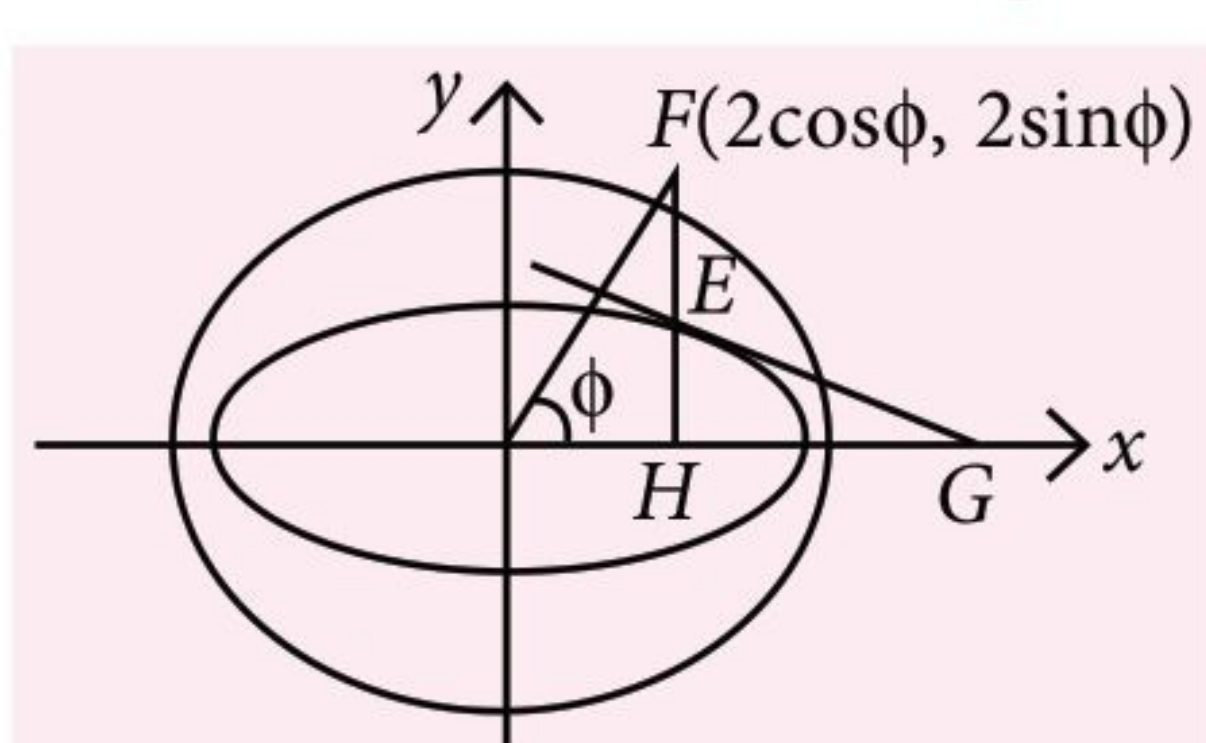
Hence, the system has no solution.

(IV) If $\frac{p}{q} = 10$, then again the system has infinitely

many solutions.

[Rating : Challenging]

18. (c) :



$E(2\cos\phi, \sqrt{3}\sin\phi)$

The equation of EG : $\frac{x}{2}\cos\phi + \frac{y}{\sqrt{3}}\sin\phi = 1$

Hence $G \equiv \left(\frac{2}{\cos\phi}, 0\right)$ and $H \equiv (2\cos\phi, 0)$

$$ar(\Delta FGH) = \frac{1}{2} HG \times FH = \frac{1}{2} \cdot \left(\frac{2}{\cos\phi} - 2\cos\phi\right) \cdot 2\sin\phi = \frac{2\sin^3\phi}{\cos\phi}$$

$$\text{So, } f(\phi) = \frac{2\sin^3\phi}{\cos\phi}. \text{ Now, } f\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\left(\frac{1}{\sqrt{2}}\right)^3}{\frac{1}{\sqrt{2}}} = 1,$$

$$f\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\left(\frac{\sqrt{3}}{2}\right)^3}{\left(\frac{1}{2}\right)} = \frac{3\sqrt{3}}{2}, \quad f\left(\frac{\pi}{6}\right) = 2 \cdot \frac{\left(\frac{1}{2}\right)^3}{\frac{\sqrt{3}}{2}} = \frac{1}{2\sqrt{3}}$$

$$\text{and } f\left(\frac{\pi}{12}\right) = 2 \cdot \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^3}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)} = \frac{(\sqrt{3}-1)^4}{8}$$

[Rating : Medium]

PAPER-2

1. (1) : Let $P = \frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}$

$$= \frac{\cos(\alpha-\beta)}{\cos\beta\sin\beta} + \frac{\cos(\alpha-\beta)}{\sin\alpha\cos\alpha}$$

$$= 2\cos(\alpha-\beta) \left\{ \frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha} \right\}$$

$$= \frac{2\cos(\alpha-\beta) \cdot 2\sin(\alpha+\beta)\cos(\alpha-\beta) \cdot 2}{\cos 2(\alpha-\beta) - \cos 2(\alpha+\beta)}$$

$$= \frac{8\cos(\alpha-\beta)\sin(\alpha+\beta) \cdot \cos(\alpha-\beta)}{2\cos^2(\alpha-\beta) - 1 - 1 + 2\sin^2(\alpha+\beta)}$$

$$= \frac{4\cos^2(\alpha-\beta)\sin(\alpha+\beta)}{\cos^2(\alpha-\beta) + \sin^2(\alpha+\beta) - 1} = \frac{4 \cdot \frac{4}{9} \cdot \frac{1}{3}}{\frac{4}{9} + \frac{1}{9} - 1} = \frac{\frac{16}{9} \cdot \frac{1}{3}}{-\frac{4}{9}} = -\frac{4}{3}$$

$$E = p^2 = \frac{16}{9}$$

$$\therefore [E] = 1$$

[Rating : Medium]

2. (8) : $xdy - (y^2 - 4y)dx = 0, x > 0$

$$\therefore \int \frac{dy}{y^2 - 4y} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{4} \ln \left| \frac{y-4}{y} \right| = \ln|x| + c \Rightarrow \left(\frac{y-4}{y} \right) = x^4 c$$

$$\Rightarrow y - 4 = yx^4 c \Rightarrow y(1 - cx^4) = 4$$

$$\text{At } y(1) = 2 \Rightarrow 2(1 - c) = 4 \Rightarrow c = -1$$

$$y = \frac{4}{1+x^4} \Rightarrow y(\sqrt{2}) = \frac{4}{5}$$

$$\therefore 10y(\sqrt{2}) = 8$$

[Rating : Easy]

3. (5) : Let $y = f(x) = \log_2(x^3 + 1)$

$$\Rightarrow 2^y - 1 = x^3 \Rightarrow x = (2^y - 1)^{1/3} = f^{-1}(y)$$

$$\text{Now } I = \int_1^2 \log_2(x^3 + 1) dx + \int_1^{\log_2 9} (2^x - 1)^{1/3} dx$$

$$= \int_1^2 f(x) dx + \int_1^{\log_2 9} f^{-1}(x) dx = 2 \log_2 9 - 1$$

$$\text{Now } 2^3 < 9 < 2^{7/2} \Rightarrow 3 < \log_2 9 < \frac{7}{2} \Rightarrow 5 < 2 \log_2 9 - 1 < 6$$

$$\text{Hence, } [I] = 5$$

[Rating : Medium]

4. (1) : We have, $x^{(16(\log_5 x)^3 - 68\log_5 x)} = 5^{-16}$

Taking log both sides, we get

$$\log_5 x = t \Rightarrow t(16t^3 - 68t) = -16$$

$$\Rightarrow 16t^4 - 68t^2 + 16 = 0 \Rightarrow 4t^4 - 17t^2 + 4 = 0$$

$$\therefore t_1 + t_2 + t_3 + t_4 = 0$$

$$\Rightarrow \log_5 x_1 + \log_5 x_2 + \log_5 x_3 + \log_5 x_4 = 0$$

$$\Rightarrow x_1 x_2 x_3 x_4 = 1$$

[Rating : Medium]

Thus, $6\beta = 5$

6. (3): $A = \begin{bmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{bmatrix}$

We have, $|A| = 1(-2\beta - 3) - 1(-2\beta - 2) = -1$

Since, $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix. So

$$|A^5| |(A^2 - (\beta - 1)A - \beta I)| = 0 \Rightarrow |A^5| |(A + I)(A - \beta I)| = 0$$

$$\text{Now, } A + I = \begin{bmatrix} \beta + 1 & 0 & 1 \\ 2 & 2 & -2 \\ 3 & 1 & -1 \end{bmatrix}$$

$$|A + I| = 2[-\beta - 1 - 3] + 2(\beta + 1) + 2 = -4$$

$$A - \beta I = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 - \beta & -2 \\ 3 & 1 & -2 - \beta \end{bmatrix}$$

$$|A - \beta I| = 2 - 3(1 - \beta) = 3\beta - 1$$

Now, $|A - \beta I| = 0 \Rightarrow 3\beta - 1 = 0$

$$\therefore \beta = \frac{1}{3}. \text{ Thus, } 9\beta = 3$$

7. (7) : $PN = \delta$

$$S_1 P = \beta$$

We have, $S_1P - SP = 20$

$$\Rightarrow \beta - \frac{\delta}{\sin \frac{\alpha}{2}} = 20$$

$$\Rightarrow \beta^2 - \frac{2\beta\delta}{\sin \frac{\alpha}{2}} + \frac{\delta^2}{\sin^2 \frac{\alpha}{2}} = 400$$

$$\begin{aligned} \text{Also, } \cos \alpha &= \frac{SP^2 + \beta^2 - 656}{2\beta \frac{\delta}{\sin \frac{\alpha}{2}}} = \frac{\frac{2\beta\delta}{\sin \frac{\alpha}{2}} - 256}{\frac{2\beta\delta}{\sin \frac{\alpha}{2}}} \\ \Rightarrow \frac{2\beta\delta}{\sin \frac{\alpha}{2}} (1 - \cos \alpha) &= 256 \Rightarrow \beta\delta \cdot 2 \sin \frac{\alpha}{2} = 128 \\ \text{Thus, } \frac{\beta\delta \sin \frac{\alpha}{2}}{9} &= \frac{64}{9} \Rightarrow \left[\frac{\beta\delta}{9} \sin \frac{\alpha}{2} \right] = 7, \end{aligned}$$

where $[\cdot]$ denotes greatest integer function.

8. (6): $f(x) = x^2 + \frac{5}{12}$

$$g(x) = \begin{cases} 2\left(1 - \frac{4(x)}{3}\right) & |x| \leq \frac{3}{4} \\ 0 & |x| > \frac{3}{4} \end{cases}$$

Solving $f(x)$ and $g(x)$, we have

$$x^2 + \frac{5}{12} = 2 \left(1 - \frac{4|x|}{3} \right)$$

$$\Rightarrow |x|^2 + \frac{8|x|}{3} - \frac{19}{12} = 0$$

$$\Rightarrow 12|x|^2 + 32|x| - 19 = 0$$

$$\Rightarrow 12|x|^2 + 38|x| - 6|x| - 19 = 0$$

$$\Rightarrow (6|x| + 19)(2|x| - 1) = 0 \Rightarrow 2|x| - 1 = 0$$

$$\Rightarrow |x| = \frac{1}{2} \Rightarrow x = \frac{1}{2}$$

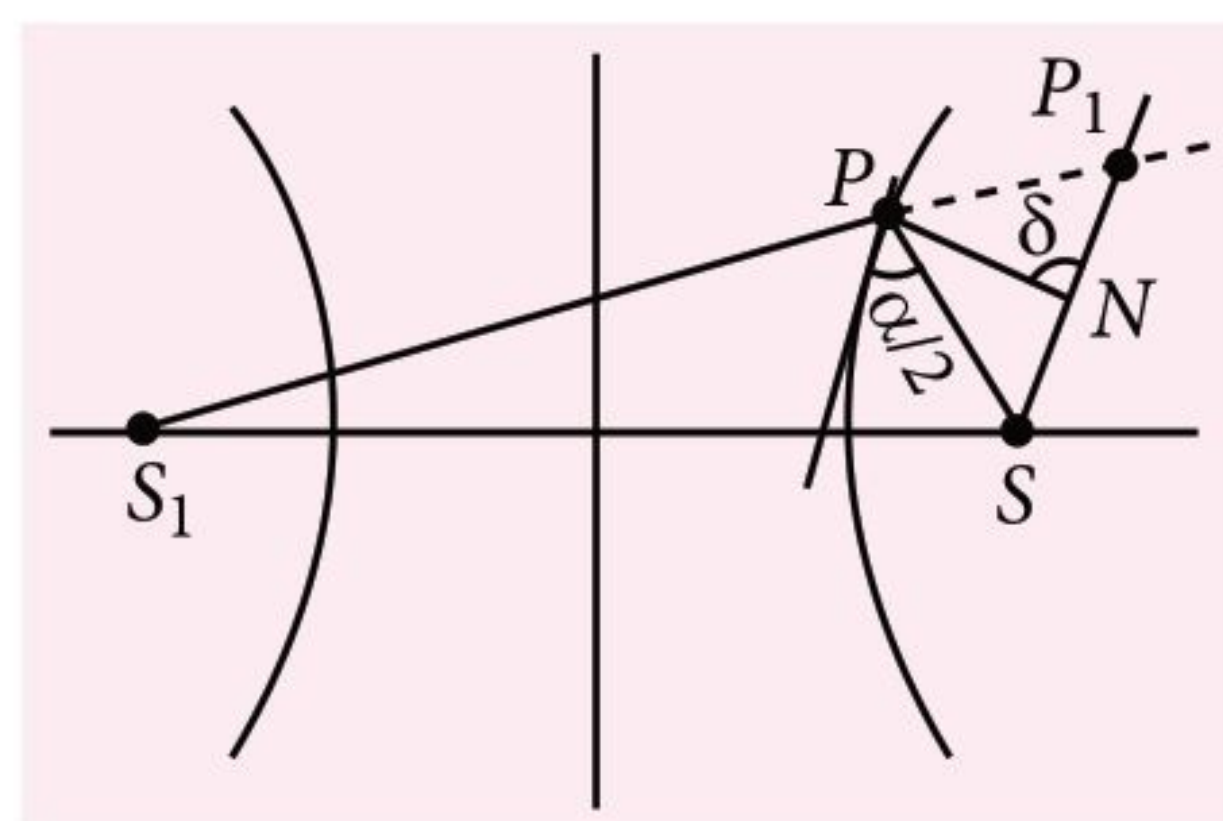
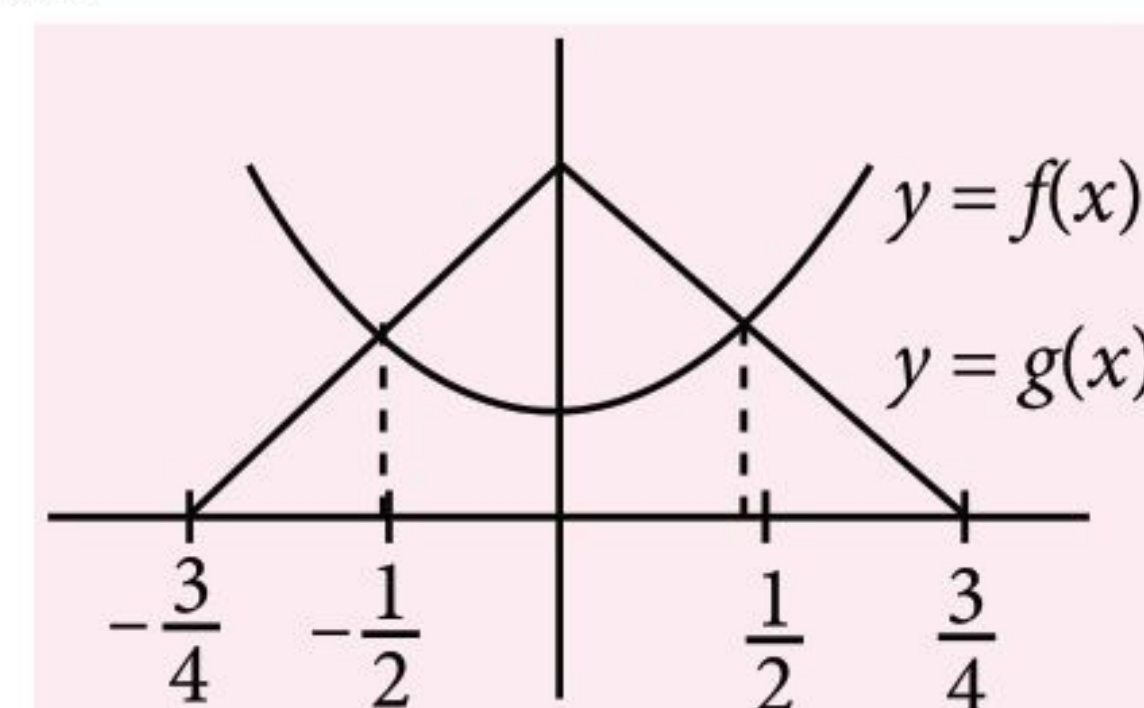
$$\text{Now, } \alpha = 2 \int_0^{1/2} \left(x^2 + \frac{5}{12} \right) dx + 2 \int_{1/2}^{3/4} \left(2 - \frac{8x}{3} \right) dx$$

$$\begin{aligned} &= 2 \left[\frac{x^3}{3} + \frac{5}{12}x \right]_0^{1/2} + 4 \left[x - \frac{2x^2}{3} \right]_{1/2}^{3/4} \\ &= 2 \left(\frac{1}{24} + \frac{5}{24} \right) + 4 \left(\frac{3}{4} - \frac{3}{8} - \frac{1}{2} + \frac{1}{6} \right) \\ &= 2 \cdot \frac{6}{24} + 4 \cdot \left(\frac{18 - 9 - 12 + 4}{24} \right) = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \end{aligned}$$

Thus, $9\delta = 6$

9. (a, b) : Applying sine rule in ΔPQR ,

$$\frac{\alpha}{\sin 30^\circ} = \frac{1}{\sin 80^\circ} \Rightarrow \alpha = \frac{1}{2 \sin 80^\circ}$$



Applying sine rule in ΔPRS

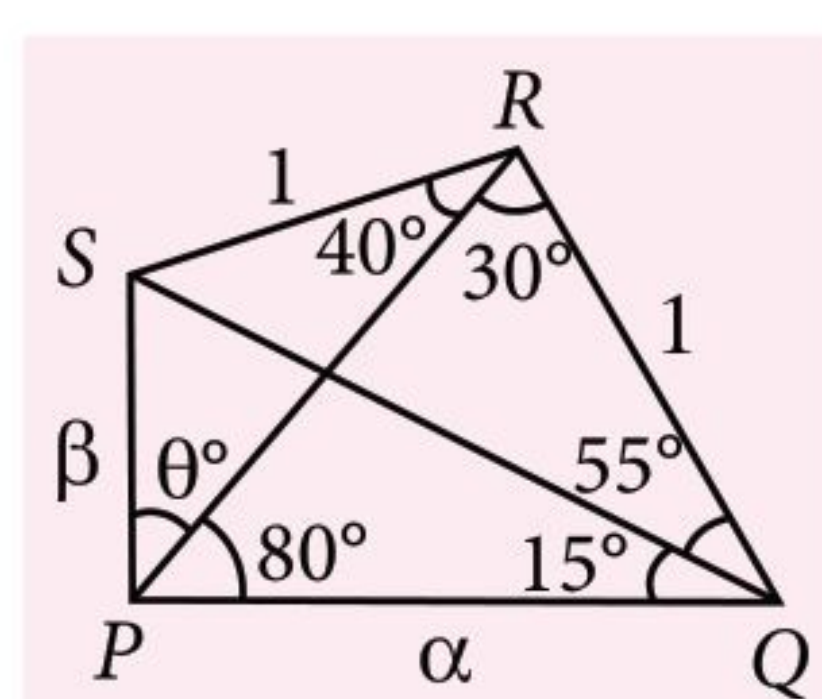
$$\frac{\beta}{\sin 40^\circ} = \frac{1}{\sin \theta^\circ}$$

$$\Rightarrow \beta \sin \theta^\circ = \sin 40^\circ$$

$$\text{Now } 4\alpha\beta \sin \theta = \frac{4 \sin 40^\circ}{2 \sin 80^\circ} = \sec 40^\circ$$

$$\therefore \sec 30^\circ < \sec 40^\circ < \sec 45^\circ$$

$$\text{Thus, } \sec 40^\circ \in \left(\frac{2}{\sqrt{3}}, \sqrt{2} \right)$$



$$10. (a, b, c) : \alpha = \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right)^4 + \dots = \frac{\left(\frac{1}{2} \right)^2}{1 - \frac{1}{4}} = \frac{1}{3}$$

$$\therefore g(x) = 2^{x/3} + 2^{(1-x)/3} = 2^{x/3} + \frac{2^{1/3}}{2^{x/3}},$$

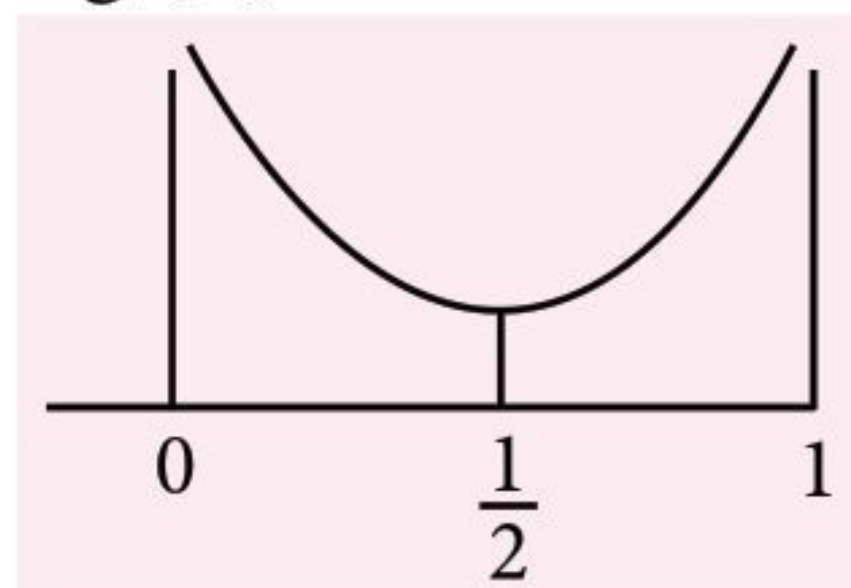
$$\text{where } g(0) = 1 + 2^{1/3} \text{ and } g(1) = 1 + 2^{1/3}$$

By A.M.-G.M. inequality, we have $g(x) \geq 2\sqrt{2^{1/3}}$ and its minimum value is attained, when $g'(x) = 0$

$$\Rightarrow x = \frac{1}{2}$$

The graph of $g'(x)$ is as shown here.

$$\text{and } g(1/2) = 2^{7/6}$$



[Rating : Challenging]

$$11. (a) : \text{Let } \bar{z}^2 + \frac{1}{z^2} = a + ib ; a, b \in \mathbb{Z}$$

$$\text{then } z^2 + \frac{1}{\bar{z}^2} = a - ib ; a, b \in \mathbb{Z}$$

Multiplying, we get

$$(z\bar{z})^2 + \frac{1}{(z\bar{z})^2} + 2 = a^2 + b^2 \Rightarrow 4 + \frac{1}{4} + 2 = a^2 + b^2,$$

$$\text{when } 4 = |z|^4$$

Now we check each option.

$$\text{For } u = \frac{43 + 3\sqrt{205}}{2} ; u + \frac{1}{u} + 2 = 45 = a^2 + b^2$$

$$\Rightarrow a = \pm 6, b = \pm 3$$

$$\text{For } u = \frac{7 + \sqrt{33}}{4} ; u + \frac{1}{u} + 2 = \frac{11}{2}$$

$$\text{For } u = \frac{9 + \sqrt{65}}{4} ; u + \frac{1}{u} + 2 = \frac{13}{2}$$

$$\text{For } u = \frac{7 + \sqrt{13}}{6} ; u + \frac{1}{u} + 2 = \frac{13}{2}$$

Thus, only option (a) is correct.

[Rating : Challenging]

$$12. (c, d) : \text{We have, } \sin \frac{\pi}{n} = \frac{r}{R+r}$$

$$\Rightarrow 1 + \frac{R}{r} = \operatorname{cosec} \frac{\pi}{n}$$

$$\text{Thus, } \frac{r}{R} = \frac{1}{\operatorname{cosec} \frac{\pi}{n} - 1}$$

$$\text{For } n = 4; \frac{r}{R} = \frac{1}{\sqrt{2} - 1}$$

$$\text{For } n = 5, \frac{r}{R} = \frac{1}{\operatorname{cosec} \frac{\pi}{5} - 1} > \frac{1}{\operatorname{cosec} \frac{\pi}{6} - 1} \Rightarrow r > R$$

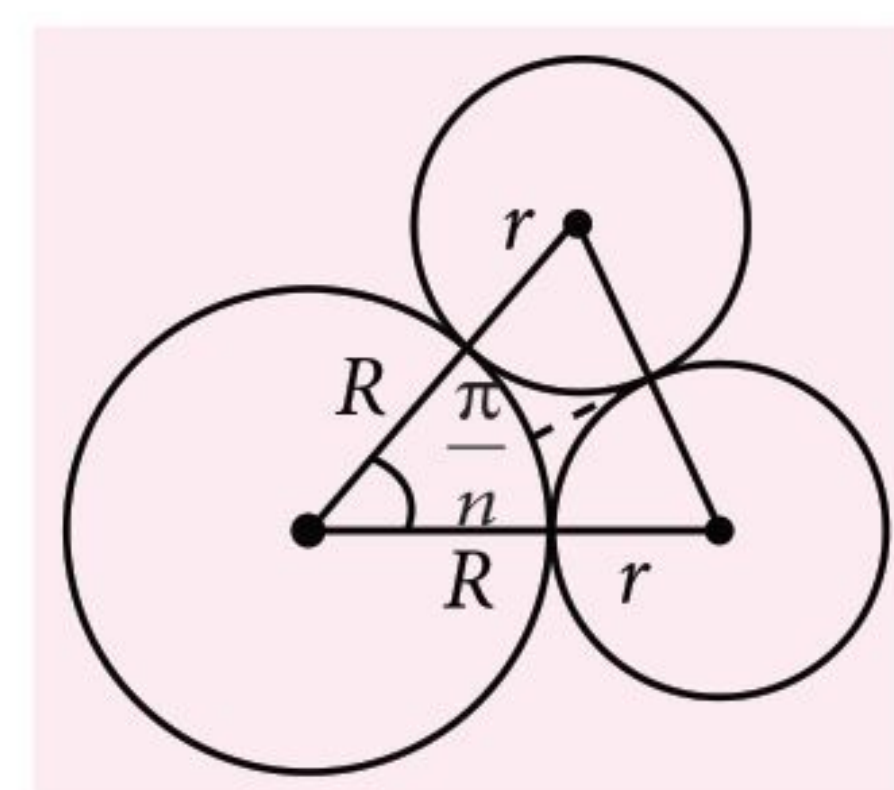
$$\text{For } n = 8, \frac{r}{R} = \frac{1}{\operatorname{cosec} \frac{\pi}{8} - 1} < \frac{1}{\operatorname{cosec} \frac{\pi}{4} - 1} \Rightarrow r < \frac{R}{\sqrt{2} - 1}$$

$$\text{For } n = 12, \frac{r}{R} = \frac{1}{\operatorname{cosec} \frac{\pi}{12} - 1} = \frac{1}{\frac{2\sqrt{2}}{\sqrt{3} - 1} - 1}$$

$$= \frac{1}{\sqrt{2}(\sqrt{3} + 1) - 1} > \frac{1}{\sqrt{2}(\sqrt{3} + 1)}$$

$$\Rightarrow \sqrt{2}(\sqrt{3} + 1)r > R$$

[Rating : Challenging]



$$13. (b, c, d) : \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 3 - c_1 \\ 1 - c_2 \\ -1 - c_3 \end{bmatrix}$$

$$\text{We have, } b_2c_3 - b_3c_2 = c_1 - 3;$$

$$c_3 - b_3c_1 = 1 - c_2;$$

$$\text{and } c_2 - b_2c_1 = 1 + c_3$$

$$\vec{b} \times \vec{c} = \hat{i}(b_2c_3 - b_3c_2) - \hat{j}(c_3 - b_3c_1) + \hat{k}(c_2 - b_2c_1)$$

$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - 3\hat{i} - \hat{j} + \hat{k} = \vec{c} - \vec{a}. \text{ So, } \vec{b} \times \vec{c} = \vec{c} - \vec{a}$$

$$\text{Taking dot product with } \vec{b}, \text{ we have } 0 = \vec{c} \cdot \vec{b} - \vec{a} \cdot \vec{b}$$

$$\Rightarrow \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{b} = 0 \therefore \vec{b} \cdot \vec{c} = 0$$

Taking dot product with $\vec{c}$, we get

$$0 = |\vec{c}|^2 - \vec{a} \cdot \vec{c} \Rightarrow |\vec{c}|^2 = \vec{a} \cdot \vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{c} \neq 0 \text{ (as } \vec{c} \cdot \vec{c} = |\vec{c}|^2 \neq 0). \text{ Again } \vec{b} \times \vec{c} = \vec{c} - \vec{a}$$

$$\text{Squaring both sides, we get } |\vec{b}|^2 |\vec{c}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a}$$

$$\Rightarrow |\vec{b}|^2 |\vec{c}|^2 = |\vec{c}|^2 + 11 - 2|\vec{c}|^2 \Rightarrow |\vec{b}|^2 |\vec{c}|^2 = 11 - |\vec{c}|^2$$

$$\Rightarrow |\vec{c}|^2 (|\vec{b}|^2 + 1) = 11 \Rightarrow |\vec{c}|^2 = \frac{11}{|\vec{b}|^2 + 1} \Rightarrow |\vec{c}| \leq \sqrt{11}$$

Again, $b_2 - b_3 = -3$

$$\Rightarrow b_2^2 + b_3^2 - 2b_2b_3 = 9 \text{ (As } b_2b_3 > 0)$$

$$\Rightarrow b_2^2 + b_3^2 = 9 + 2b_2b_3 > 9$$

$$\therefore |\vec{b}| = \sqrt{1 + b_2^2 + b_3^2} > \sqrt{10} \quad [\text{Rating : Challenging}]$$

14. (c) : $\frac{dy}{dx} + 12y = \cos\left(\frac{\pi}{12}x\right)$,

which is a linear differential equation.

$$\text{I.F} = e^{\int 12 dx} = e^{12x}$$

Solution is given by, $ye^{12x} = \int e^{12x} \cos\left(\frac{\pi}{12}x\right) dx$

$$= \frac{e^{12x}}{12^2 + \left(\frac{\pi}{12}\right)^2} \left\{ 12 \cos \frac{\pi}{12}x + \frac{\pi}{12} \sin \frac{\pi}{12}x \right\} + C$$

$$\Rightarrow y = \frac{12}{12^4 + \pi^2} \left\{ 12^2 \cos \frac{\pi x}{12} + \pi \sin \frac{\pi x}{12} \right\} + \frac{C}{e^{12x}}$$

Given $y(0) = 0$

$$\Rightarrow 0 = \frac{12}{12^4 + \pi^2} (12^2 + 0) + C \Rightarrow C = \frac{-12^3}{12^4 + \pi^2}$$

Now, $\frac{dy}{dx} = \frac{12}{12^4 + \pi^2}$

$$\left\{ -12\pi \sin \frac{\pi x}{12} + \frac{\pi^2}{12} \cos \frac{\pi x}{12} + 1728e^{-12x} \right\}$$

For $x \leq 0$, $\frac{dy}{dx} > 0$. But for $x > 0$, $\frac{dy}{dx}$ may be negative

or positive.

So $f(x)$ is neither increasing nor decreasing.

For some $\beta \in R$, $y = \beta$ intersects $y = f(x)$ at infinitely many points. [Rating : Challenging]

15. (a) : $10 = 2 + 2 + 2 + 4 = 2 + 2 + 3 + 3$

In the first case the number of ways

$$= {}^4C_1 \cdot ({}^3C_1 \cdot {}^2C_1)^3 \cdot {}^5C_4 = 4320$$

In the second case the number of ways

$$= {}^4C_2 \cdot ({}^3C_1 \cdot {}^2C_1)^2 \cdot ({}^5C_3 - 1)^2 = 6.6^2 \times 9^2 = 216 \times 81 = 17496$$

Total number of ways = $17496 + 4320$

$$= 21816$$

[Rating : Challenging]

16. (a) : $M = \begin{bmatrix} 5/2 & 3/2 \\ -3/2 & -1/2 \end{bmatrix}$

M satisfies $M^2 - \left(\frac{5}{2} - \frac{1}{2}\right)M + \left(-\frac{5}{4} + \frac{9}{4}\right)I = 0$

$$\Rightarrow M^2 - 2M + I = 0 \Rightarrow \text{i.e. } (M - I)^2 = 0$$

Let $M - I = B$, $B^2 = 0$. Now, $M = I + B$

$$M^n = (I + B)^n = I + {}^nC_1 B + {}^nC_2 B^2 + \dots$$

$$= I + nB = I + n(M - I)$$

$$M^{2022} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2022 \begin{bmatrix} 3/2 & 3/2 \\ -3/2 & -3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3033 & 3033 \\ -3033 & -3033 \end{bmatrix} = \begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$$

[Rating : Challenging]

17. (c) : Red Blue Green Yellow Orange White

Box-I	8	3	5			
Box-II	24	9	15			
Box-III		1	12	3		
Box-IV			10		16	6

Let A : One of the chosen ball is white.

B : At least one of chosen balls is green.

We want $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{\frac{5}{16} \cdot \frac{6}{32}}{\frac{5}{16} \cdot 1 + \frac{8}{16} \cdot \frac{15}{48} + \frac{3}{16} \cdot \frac{12}{16}} = \frac{15}{156} = \frac{5}{52}$$

[Rating : Challenging]

18. (b) : $f(n) = n + \sum_{r=1}^n \frac{16r + (9 - 4r)n - 3n^2}{4rn + 3n^2}$

$$= n + \sum_{r=1}^n \frac{(16r + 9n) - (4rn + 3n^2)}{4rn + 3n^2}$$

$$= n + \left(\sum_{r=1}^n \frac{16r + 9n}{4rn + 3n^2} \right) - n$$

$$\lim_{n \rightarrow \infty} f(n) = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{16r + 9n}{4rn + 3n^2} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{\left(16 \frac{r}{n} + 9\right)}{4 \frac{r}{n} + 3}$$

$$= \int_0^1 \frac{16x + 9}{4x + 3} dx = \int_0^1 4 dx - 3 \int_0^1 \frac{dx}{4x + 3}$$

$$= 4 - 3 \cdot \left[\frac{1}{4} \log_e (4x + 3) \right]_0^1$$

$$= 4 - \frac{3}{4} \log_e 7 + \frac{3}{4} \log_e 3 = 4 - \frac{3}{4} \log_e \left(\frac{7}{3} \right)$$

[Rating : Challenging]



JEE ADVANCED 2022 RESULT

How the toppers did it, in their own words

The Indian Express spoke to the top 10 rankers of JEE Advanced this year to find that nine of them want to study computer science at IIT Bombay and only one* at IIT Madras. A look at their background and tips for preparation.



SHISHIR R K, 18

AIR
1

School : Narayana Olympiad School, Sahakarnagar (near Bengaluru)
JEE Advanced Marks : 314/360 (Phy: 96, Chem: 109, Math: 109)
Class 12 Exam : 96.4 %
Exam Prep Tip : Attempt as many mock tests as possible and note and correct errors after every exam.



POLU LAKSHMI SAI LOHITH REDDY, 17

AIR
2

School : Narayana Junior College, Hitech City, Hyderabad
JEE Advanced Marks : 307/360 (Phy: 101, Chem: 94, Math: 112)
Class 12 Exam : 97.9%
Exam Prep Tip : Focus on concepts, not on attempting more questions. Start with the NCERT book.



THOMAS BIJU CHEERAMVELIL, 18

AIR
3

School : Vishwa Prabhash Central School, Trivandrum.
JEE Advanced Marks : 300/360 (Phy: NA, Chem: NA, Math: NA)
Class 12 Exam : 99.4%
Exam Prep Tip : Study regularly and try to do many problems. Try not to repeat the mistakes.



VANGAPALLI SAI SIDDHARTHA, 18

AIR
4

School : Narayana Junior College, Hitech City, Hyderabad
JEE Advanced Marks : 299/360 (Phy: 103, Chem: 110, Math: 86)
Class 12 Exam : 98%
Exam Prep Tip : Practice math for speed. Apart from books, try to read extra materials, follow teachers.



MAYANK MOTWANI, 18

AIR
5

School : Lord Buddha Public School, Kota
JEE Advanced Marks : 297/360 (Phy: 88, Chem: 111, Math: 98)
Class 12 Exam : 98%
Exam Prep Tip : Follow the guidance of your teachers. Whatever you are taught, revise it and do all of your homework. I used to study 6-8 hours daily.



POLISETTY KARTHIKEYA, 19

AIR
6

School : Sri Chaitanya Junior College, Veda Vyasa Bhavan, Vijayawada
JEE Advanced Marks : 292/360 (Phy: 98, Chem: 103, Math: 91)
Class 12 Exam : 93.9%
Exam Prep Tip : What sets one apart is confidence. Try not to panic and attempt questions calmly.



PRATIK SAHOO, 18

AIR
7

School : Prodigy Public School, Pune
JEE Advanced Marks : 292/360 (Phy: 104, Chem: 103, Math: 85)
Class 12 Exam : 94.8%
Exam Prep Tip : Don't fall for the popular 12-hour study regime. I think 7-8 hours a day is equally good. Quality of hours matter more than the quantity.



DHEERAJ KURUKUNDA, 17

AIR
8

School : Narayana Junior College, Hitech City, Hyderabad
JEE Advanced Marks : 287/360 (Phy: 92, Chem: 107, Math: 88)
Class 12 Exam : 98.8%
Exam Prep Tip : Attempting more questions should not be the aim. Focus on the basics.



MAHIT GADHIWALA, 18

AIR
9

School : Scholar English Academy School, Surat
JEE Advanced Marks : 285/360 (Phy: 107, Chem: 93, Math: 85)
Class 12 Exam : 98.6%
Exam Prep Tip : 4-5 hours of self-study daily apart from school and coaching class.



VENCHA GNANA MAHESH, 17

AIR
10

School : Narayana Junior College, Visakhapatnam
JEE Advanced Marks : 285/360 (Phy: 86, Chem: 99, Math: 100)
Class 12 Exam : 97.6%
Exam Prep Tip : Prepare short notes for important concepts, work on time management, avoid silly mistakes during examination.

Courtesy : The Indian Express

CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

SERIES - II Matrices and Determinants

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time Allowed : 45 Minutes

Maximum Marks : 200

Multiple Choice Questions (MCQs)

- A square matrix $A = [a_{ij}]_{n \times n}$ is called a lower triangular matrix if $a_{ij} = 0$ for
 - $i = j$
 - $i < j$
 - $i > j$
 - None of these
- If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to
 - $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$
 - $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$
 - $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$
 - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$, then the values of x, y, z and w respectively are
 - 2, 2, 3, 4
 - 2, 3, 1, 2
 - 3, 3, 0, 1
 - None of these
- Let $A = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$, then $AB = O$, if
 - $\theta = n\phi, n = 0, 1, 2, \dots$
 - $\theta + \phi = n\pi, n = 0, 1, 2, \dots$
 - $\theta = \phi + (2n+1)\frac{\pi}{2}, n = 0, 1, 2, \dots$
 - $\theta = \phi + \frac{n\pi}{2}, n = 0, 1, 2, \dots$
- If $\Delta = \begin{vmatrix} Ax^2 & x^3 & 1 \\ By^2 & y^3 & 1 \\ Cz^2 & z^3 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} Ax & By & Cz \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$, then
 - $\Delta + \Delta_1 = 0$
 - $\Delta \neq \Delta_1$
 - $\Delta = x\Delta_1$
 - $\Delta - \Delta_1 = 0$
- If $AB = A$ and $BA = B$, then
 - $B = I$
 - $A = I$
 - $A^2 = A$
 - $B^2 = I$
- If $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$, then value of Δ is
 - $a^7 + b^7 + c^7$
 - $(a+b+c)^7$
 - $(a^2+b^2+c^2)(a^5+b^5+c^5)$
 - 0
- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to
 - $2I$
 - $3I$
 - $-2I$
 - null matrix
- Let $\Delta = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$, then the value of Δ is
 - $(x-y)(y-z)(z-x)$
 - xyz
 - $(x^2+y^2+z^2)^2$
 - $xyz(x-y)(y-z)(z-x)$
- If $A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$, then

- (a) $A + B = B + A$ and $A + (B + C) = (A + B) + C$
 (b) $A + B = B + A$ and $AC = BC$
 (c) $A + B = B + A$ and $AB = BC$
 (d) $AC = BC$ and $A = BC$
11. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of an equilateral triangle whose each side is equal to a , then $\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$ is equal to
 (a) $3a^4$ (b) $12a^4$ (c) $9a^4$ (d) $16a^4$
12. If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, then the value of $x + y + z$ is
 (a) 3 (b) 0 (c) 2 (d) 1
13. Find the cofactors of elements a_{12} , a_{22} , a_{32} respectively of the matrix $\begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$.
 (a) 0, 2, $-2\sin \theta$ (b) 2, 0, $2\sin \theta$
 (c) 2, 0, $-2\sin \theta$ (d) $-2\sin \theta$, 2, 0
14. The value of $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to
 (a) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ (b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$ (c) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ (d) $\begin{bmatrix} 44 \\ 45 \end{bmatrix}$
15. The cofactor of each element of the first column of matrix $A = \begin{bmatrix} g & f & c \\ h & b & f \\ a & h & g \end{bmatrix}$ is
 (a) $(bg - hf)$, $(ch - fg)$, 1
 (b) $(bg - hf)$, $(ch - fg)$, $(f^2 - bc)$
 (c) 0, 0, 0
 (d) $(bg - hf)$, $(ch - fg)$, 0
16. If $A = \begin{bmatrix} b & 2 & a \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA^T = 9I_3$, then find the values of a and b respectively.
 (a) 1, 2 (b) 2, 1 (c) -1, 2 (d) -2, 1
17. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
 (a) $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$
18. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then
 (a) $x = 0, y = 5$ (b) $x + y = 5$
 (c) $x = y$ (d) $x = 0, y = 9$
19. The matrix $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$ is singular, if
 (a) $a = 0$ (b) $a + b = 0$
 (c) $a - b = 0$ (d) $a + b + c = 0$
20. If $A = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then $x =$
 (a) 4 (b) 3 (c) -4 (d) -3
21. If the equations $a(y + z) = x$, $b(z + x) = y$, $c(x + y) = z$ have non-trivial solutions, then the value of $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$ is
 (a) 1 (b) 2 (c) -1 (d) -2
22. If $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and a skew-symmetric matrix C , then B is
 (a) $\begin{bmatrix} 1 & 5/2 & 0 \\ 5/2 & 5 & 5/2 \\ 0 & 5/2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5/2 & 0 \\ 5/2 & 5 & 5/2 \\ 0 & 5/2 & 1 \end{bmatrix}$
 (c) $\begin{bmatrix} 1 & 5/2 & 0 \\ 5/2 & 5 & 5/2 \\ 0 & 5/2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5/2 & 0 \\ 5/2 & 5 & 3/2 \\ 0 & 5/2 & 2 \end{bmatrix}$
23. Suppose that system of equations $x = cy + bz$, $y = az + cx$, $z = bx + ay$ has non-trivial solution. Then $a^2 + b^2 + c^2 + 2abc =$
 (a) 2 (b) -1 (c) 0 (d) 1
24. Find the inverse of matrix $\begin{bmatrix} 2 & 0 & 3 \\ 5 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$.
 (a) $\frac{1}{13} \begin{bmatrix} -1 & 3 & -3 \\ 5 & -2 & 15 \\ 5 & -2 & 2 \end{bmatrix}$ (b) $\frac{1}{13} \begin{bmatrix} -1 & 3 & 2 \\ -5 & 2 & 15 \\ 5 & -2 & 2 \end{bmatrix}$

$$(c) \frac{1}{23} \begin{bmatrix} 3 & -2 & 2 \\ 5 & 2 & -5 \\ 5 & -5 & 3 \end{bmatrix} \quad (d) \frac{1}{13} \begin{bmatrix} 3 & -1 & 2 \\ 5 & 5 & -5 \\ 15 & -5 & 3 \end{bmatrix}$$

25. The determinant $\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$ equals

- (a) $abc(b - c)(c - a)(a - b)$
 (b) $(b - c)(c - a)(a - b)$
 (c) $(a + b + c)(b - c)(c - a)(a - b)$
 (d) None of these

26. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If

B is the inverse of matrix A , then α is

- (a) -2 (b) 1 (c) 2 (d) 5

27. There are two values of a which makes determinant,

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86, \text{ then the sum of these numbers}$$

is

- (a) 4 (b) 5 (c) -4 (d) 9

28. If $\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x + iy$, then xy equals

- (a) 0 (b) 1 (c) 2 (d) 3

29. If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \neq 0$, then area of the triangle

whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right)$ and $\left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is

- (a) $\frac{1}{8}abc$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}abc$ (d) $\frac{1}{4}$

30. If A is a square matrix of order 3 and if $\det(A) = 3$, then $\det[\text{adj}(\text{adj}(\text{adj} A))]$ is equal to

- (a) $(81)^2$ (b) 81 (c) 729 (d) 27

31. Let $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then absolute value of

$\frac{1}{3} \det(3(M + M^T))$ is equal to

- (a) 72 (b) -72 (c) 82 (d) 78

32. If P is a 3×3 matrix such that $P' = 2P + I$, where P' is the transpose of P , then

- (a) $P = I$ (b) $P = -I$
 (c) $P = 2I$ (d) $P = -2I$

33. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{2017} is equal to

- (a) $2^{2014}A$ (b) $2^{2015}A$
 (c) $2^{2016}A$ (d) $2^{2020}A$

34. If $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ and $A + A^T = I$, where I is unit matrix of order 2×2 and A^T is the transpose of A , then value of θ is equal to

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

Assertion & Reasoning Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- (a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
 (b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
 (c) If Statement-I is true but Statement-II is false.
 (d) If Statement-I is false and Statement-II is true.

35. Let $A = \begin{bmatrix} 1 & 0 & a \\ 2 & 3 & b \\ -3 & 1 & c \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & x \\ 2 & 3 & y \\ -3 & 1 & z \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & a+x \\ 2 & 3 & b+y \\ -3 & 1 & c+z \end{bmatrix}$.

Statement-I : $\det A + \det B = \det C$.

Statement-II : $A + B = C$.

36. **Statement-I :** If $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$, then $A^3 + A^2 + A = I$.

Statement-II : If $\det(A - \lambda I) = C_0\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$, then $C_0A^3 + C_1A^2 + C_2A + C_3I = 0$

37. **Statement-I :** $B = \begin{bmatrix} -\frac{1}{2} & \sqrt{5} & 2 & 3 \end{bmatrix}_{1 \times 4}$ is a row matrix.

Statement-II : If $B = [b_{ij}]_{1 \times n}$ is a row matrix, then its order is $1 \times n$.

38. Statement-I : For multiplication of two matrices A and B , the number of columns in A should be less than the number of rows in B .

Statement-II : For getting the elements of the product matrix, we take rows of A and columns of B , multiply them elementwise and take the sum.

Matrix-Match

39. Match the column-I with column-II and select the correct answer using the options given below :

Column I		Column II	
(P)	A is a real skew-symmetric matrix such that $A^2 + I = 0$, then AA' is equal to	(1)	A
(Q)	A is a matrix such that $A^2 = A$ if $(I + A)^n = I + \lambda A$, then λ equals	(2)	0
(R)	If for a matrix A , $A^2 = A$ and $B = I - A$, then $AB + BA + I - (I - A)^2$ equals	(3)	$2^n - 1$
(S)	If $A' = A$ and $B' = B$, then $(AB - BA)' - (BA - AB)$ equals	(4)	I

- (a) P-1, Q-2, R-4, S-3 (b) P-2, Q-1, R-3, S-4
(c) P-3, Q-4, R-2, S-1 (d) P-4, Q-3, R-1, S-2

40. If $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = px^5 + qx^4 + rx^3 + sx^2 + tx + w$,

then match the following columns and select the correct answer using the option give below.

Column I		Column II	
(P)	w is equal to	(1)	3
(Q)	t is equal to	(2)	1
(R)	$p + r + w$ is equal to	(3)	-2
(S)	$q + r$ is equal to	(4)	0

- (a) P - 2, Q - 1, R - 4, S - 3
(b) P - 4, Q - 2, R - 3, S - 1
(c) P - 3, Q - 4, R - 1, S - 2
(d) P - 1, Q - 3, R - 2, S - 4

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

To promote the construction of toilets in villages, an organisation tried to generate awareness through (1) house calls (2) emails and (3) announcements. The cost for each mode per attempt is given below :



- (1) ₹ 50 (2) ₹ 20 (3) ₹ 40

The number of attempts made in the villages X , Y and Z are given below :

	(1)	(2)	(3)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given modes is

- (1) 2% (2) 4% (3) 20%

- 41.** The cost incurred by the organisation on village X is
(a) ₹ 10000 (b) ₹ 15000
(c) ₹ 30000 (d) ₹ 20000
- 42.** The cost incurred by the organisation on village Y is
(a) ₹ 25000 (b) ₹ 18000
(c) ₹ 23000 (d) ₹ 28000
- 43.** The cost incurred by the organisation on village Z is
(a) ₹ 19000 (b) ₹ 39000
(c) ₹ 45000 (d) ₹ 50000
- 44.** The total number of toilets that can be expected after the promotion in village X , is
(a) 20 (b) 30 (c) 40 (d) 50
- 45.** The total number of toilets that can be expected after the promotion in village Z , is
(a) 26 (b) 36 (c) 46 (d) 56

Case II : Read the following passage and answer the questions from 46 to 50.

Three shopkeepers Salim, Vijay and Venket are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepers Salim, Vijay and Venket are using (20, 30, 40), (30, 40, 20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Salim, Vijay and Venket spent ₹ 250, ₹ 270 and ₹ 200 on these carry bags respectively.



Using the concept of matrices and determinants, answer the following questions.

46. What is the cost of one polythene bag?
(a) ₹ 1 (b) ₹ 2 (c) ₹ 3 (d) ₹ 5
47. What is the cost of one handmade bag?
(a) ₹ 1 (b) ₹ 2 (c) ₹ 3 (d) ₹ 5
48. What is the cost of one newspaper envelope?
(a) ₹ 1 (b) ₹ 2 (c) ₹ 3 (d) ₹ 5
49. Keeping in mind the social conditions, which shopkeeper is better?
(a) Salim (b) Vijay
(c) Venket (d) None of these
50. Keeping in mind the environmental conditions, which shopkeeper is better?
(a) Salim (b) Vijay
(c) Venket (d) None of these

SOLUTIONS

1. (b): $A = [a_{ij}]_{n \times n}$ is a lower triangular matrix iff all entries above the diagonal vanish, i.e., $a_{ij} = 0$ for $i < j$.

2. (b): Since, $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$$\therefore PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix} \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

3. (a): Since, $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$

$$\Rightarrow x+y=4 \dots(i), x-y=0 \dots(ii), 2x+z=7 \dots(iii) \text{ and } 2z+w=10 \dots(iv)$$

On solving these equations, we get
 $x=2, y=2, z=3$ and $w=4$

4. (c):

$$AB = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta \cos^2 \phi + \sin \theta \cos \phi \cos \theta \sin \phi \\ \cos^2 \phi \cos \theta \sin \theta + \sin^2 \theta \sin \phi \cos \phi \end{bmatrix}$$

$$\begin{bmatrix} \cos^2 \theta \sin \phi \cos \phi + \sin^2 \phi \sin \theta \cos \theta \\ \cos \theta \sin \theta \sin \phi \cos \phi + \sin^2 \theta \sin^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta \cos \phi \cos(\theta - \phi) & \sin \phi \cos \theta \cos(\theta - \phi) \\ \sin \theta \cos \phi \cos(\theta - \phi) & \sin \theta \sin \phi \cos(\theta - \phi) \end{bmatrix}$$

$$\therefore AB = O$$

$$\Rightarrow \cos(\theta - \phi) = 0 \Rightarrow \cos(\theta - \phi) = \cos(2n+1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{2} + \phi, \text{ where } n = 0, 1, 2, \dots$$

5. (d)

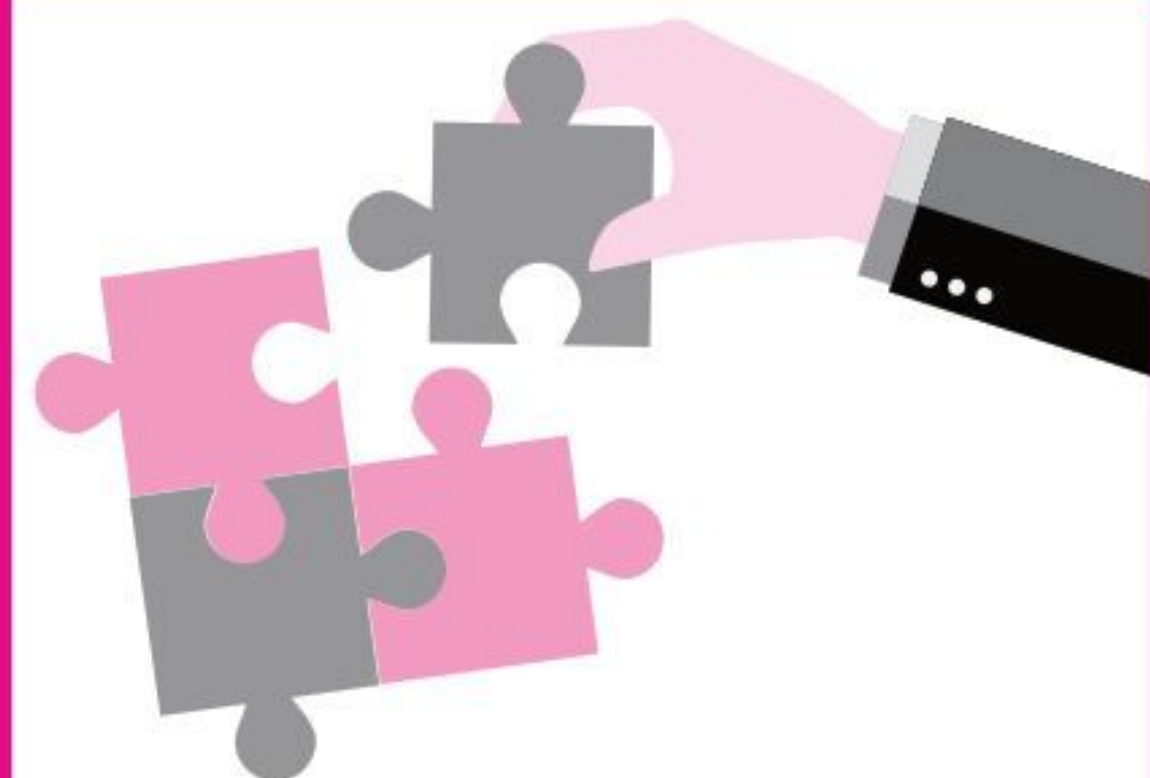
6. (c): $A = AB = A(BA) = (AB)A = A \cdot A = A^2$
 $B = BA = B(AB) = (BA)B = B \cdot B = B^2$

7. (d)

8. (a): Given that, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6×6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

9+	7+		7+		20+
	2÷		14+		
		2÷			
1-			25×	2÷	
10+				1-	
1-		6×		3-	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$\text{and } 5A = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$\therefore A^2 - 5A = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$$

9. (d): We have, $\Delta = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$

Taking x , y and z common from C_1 , C_2 and C_3 respectively, we get

$$\Delta = xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = xyz \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{vmatrix}$$

Taking $(y-x)$ common from C_2 and $(z-x)$ from C_3 , we get

$$\Delta = xyz(y-x)(z-x) \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$$

Expanding along R_1 , we get

$$\Delta = xyz(y-x)(z-x)(z+x-y-x)$$

$$\Rightarrow \Delta = xyz(x-y)(y-z)(z-x)$$

10. (a): In option (a), there are two laws, commutative law and associative law, which are satisfied by all matrices. Thus, option (a) is correct.

11. (b): Let Δ be the area of triangle ABC . Then,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow 2\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow 8\Delta = 4 \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}$$

$$\Rightarrow 64\Delta^2 = \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2 \quad \dots(i)$$

But, the area of an equilateral triangle with each side a

is $\frac{\sqrt{3}}{4}a^2$.

$$\therefore \Delta = \frac{\sqrt{3}}{4}a^2 \Rightarrow 16\Delta^2 = 3a^4 \quad \dots(ii)$$

From (i) and (ii), we get

$$\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2 = 12a^4$$

12. (a): Given, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$

$$= \frac{1}{40} \begin{bmatrix} 25+0-25 \\ -25+0+65 \\ 50+0+30 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow x = 0, y = 1 \text{ and } z = 2$$

$$\therefore x + y + z = 0 + 1 + 2 = 3$$

13. (a): $M_{12} = \begin{vmatrix} -\sin\theta & \sin\theta \\ -1 & 1 \end{vmatrix} = -\sin\theta + \sin\theta = 0$

$$\Rightarrow c_{12} = -M_{12} = 0$$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1+1=2 \Rightarrow c_{22} = M_{22} = 2$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ -\sin\theta & \sin\theta \end{vmatrix} = \sin\theta + \sin\theta = 2\sin\theta$$

$$\Rightarrow c_{32} = -M_{32} = -2\sin\theta$$

14. (a): $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 21+4+10 \\ 27+8+5 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}$$

15. (b): $M_{11} = \begin{vmatrix} b & f \\ h & g \end{vmatrix} = bg - hf \Rightarrow c_{11} = M_{11} = bg - hf$

Recipe for Success

“Study while others are sleeping; work while others are loafing; prepare while others are playing; and dream while others are wishing.”

William A. Ward

An American Motivational Writer

$$M_{21} = \begin{vmatrix} f & c \\ h & g \end{vmatrix} = fg - ch \Rightarrow c_{21} = -M_{21} = ch - fg$$

$$M_{31} = \begin{vmatrix} f & c \\ b & f \end{vmatrix} = f^2 - bc \Rightarrow c_{31} = M_{31} = f^2 - bc$$

16. (b): We have,

$$A = \begin{bmatrix} b & 2 & a \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} b & 2 & -2 \\ 2 & 1 & 2 \\ a & -2 & -1 \end{bmatrix}$$

$$\therefore AA^T = 9I_3$$

$$\Rightarrow \begin{bmatrix} b & 2 & a \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \begin{bmatrix} b & 2 & -2 \\ 2 & 1 & 2 \\ a & -2 & -1 \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} b^2 + 4 + a^2 & 2b + 2 - 2a & -2b + 4 - a \\ 2b + 2 - 2a & 9 & 0 \\ -2b + 4 - a & 0 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Comparing, we get

$$2b + 2 - 2a = 0 \text{ and } -2b + 4 - a = 0$$

Solving these equation, we get

$$a = 2, b = 1$$

17. (b): We have, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\therefore \text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

18. (c): We have, $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

$$\text{Now, } A = A^T \Rightarrow \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix} \Rightarrow x = y$$

19. (d): Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we get

$$\Delta = (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (a+b+c)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix} = (a+b+c)^3.$$

Since, the given matrix is singular

$$\therefore (a+b+c)^3 = 0 \Rightarrow a+b+c = 0$$

20. (c): A is a symmetric matrix $\Rightarrow A^T = A$

$$\Rightarrow \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$$

$$\Rightarrow x-1 = 2x+3 \Rightarrow x = -4$$

21. (b): $\begin{vmatrix} -1 & a & a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$\begin{vmatrix} -1 & a+1 & a+1 \\ b & -(b+1) & 0 \\ c & 0 & -(1+c) \end{vmatrix} = 0$$

Taking $(a+1)$, $(b+1)$ and $(c+1)$ common from R_1, R_2 and R_3 respectively.

$$\begin{vmatrix} -\frac{1}{a+1} & 1 & 1 \\ \frac{b}{b+1} & -1 & 0 \\ \frac{c}{c+1} & 0 & -1 \end{vmatrix} = 0$$

Expanding along C_1 , we get

$$-\frac{1}{a+1} + \frac{b}{b+1} + \frac{c}{c+1} = 0$$

$$\Rightarrow -\frac{1}{a+1} + 1 - \frac{1}{b+1} + 1 - \frac{1}{c+1} = 0$$

$$\therefore \frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} = 2$$

22. (b)

23. (d): $\begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

Expanding the determinant, we get

$$-1(1-a^2) - c(-c-ab) + b(ca+b) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1.$$

24. (a): Let $A = \begin{bmatrix} 2 & 0 & 3 \\ 5 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

We have $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 3 \\ 5 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$, we get

$$\begin{bmatrix} 1 & 0 & 3/2 \\ 5 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$, we get

$$\begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & -15/2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -5/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & -15/2 \\ 0 & 0 & 13/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -5/2 & 1 & 0 \\ 5/2 & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow \left(\frac{2}{13}\right)R_3$, we get

$$\begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & -15/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ -5/2 & 1 & 0 \\ 5/13 & -2/13 & 2/13 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \left(\frac{-3}{2}\right)R_3, R_2 \rightarrow R_2 + \left(\frac{15}{2}\right)R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/13 & 3/13 & -3/13 \\ 5/13 & -2/13 & 15/13 \\ 5/13 & -2/13 & 2/13 \end{bmatrix} A$$

$$\text{Hence, } A^{-1} = \frac{1}{13} \begin{bmatrix} -1 & 3 & -3 \\ 5 & -2 & 15 \\ 5 & -2 & 2 \end{bmatrix}$$

25. (d)

26. (d): Since, B is the inverse of matrix A .

$$\text{So, } 10A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow 10A^{-1} \cdot A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow 10I = \begin{bmatrix} 4+4+2 & -4+2+2 & 4-6+2 \\ -5+0+\alpha & 5+0+\alpha & -5+0+\alpha \\ 1-4+3 & -1-2+3 & 1+6+3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow -5 + \alpha = 0 \Rightarrow \alpha = 5$$

$$\mathbf{27. (c):} \Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$

Applying $R_2 \rightarrow R_2 - 2R_1$, we get

$$\begin{vmatrix} 1 & -2 & 5 \\ 0 & a+4 & -11 \\ 0 & 4 & 2a \end{vmatrix} = 86$$

Expanding along C_1 , we get

$$1[(a+4)2a - (4)(-11)] = 86$$

$$\Rightarrow 2a^2 + 8a + 44 = 86 \Rightarrow a^2 + 4a - 21 = 0$$

$$\Rightarrow (a+7)(a-3) = 0 \Rightarrow a = 3, -7$$

Now, sum of values of a is $3 + (-7) = 3 - 7 = -4$.

$$\mathbf{28. (a):} \text{ We have, } \begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = x + iy$$

$$\Rightarrow 3i(9i^2 + 9) + 9i(2i + 10) + 1(18 - 90i) = x + iy$$

$$\Rightarrow 3i(-9 + 9) + 18i^2 + 90i + 18 - 90i = x + iy$$

$$\Rightarrow 0 + 0i = x + iy \Rightarrow x = y = 0 \therefore xy = 0$$

29. (b)

30. (a): We know that, $|\text{adj}(\text{adj}(\text{adj} A))| = |A|^{(n-1)^3}$, where n is the order of matrix A .

$$= 3^{(3-1)^3} = 3^{2^3} = 3^8 = (81)^2$$

$$\mathbf{31. (a):} \text{ Given, } M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow M^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\therefore$

$$M + M^T = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} \text{ and } 3(M + M^T) = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 6 & 0 \\ 6 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } \frac{1}{3} \det(3(M + M^T)) = \frac{1}{3} \begin{vmatrix} 0 & 0 & 6 \\ 0 & 6 & 0 \\ 6 & 0 & 0 \end{vmatrix}$$

$$= \frac{6^3}{3} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 72[1(0-1)] = -72$$

$\therefore$ Absolute value of $1/3 \det(3(M + M^T))$ is 72.

32. (b) : We have, $P' = 2P + I$... (i)
 Now, $(P')' = (2P + I)' = 2P' + I$
 $\Rightarrow P = 2(2P + I) + I$ (Using (i))
 $\Rightarrow P = 4P + 3I \Rightarrow P = -I$

33. (c) : We have, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
 $\therefore A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$
 $A^3 = A^2 A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = 4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^2 A$

Similarly, $A^n = 2^{n-1} A$
 $\therefore A^{2017} = 2^{2017-1} A = 2^{2016} A$

34. (a) : Given, $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$
 $\Rightarrow A^T = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

Also, $A + A^T = I$

$$\Rightarrow \begin{bmatrix} 2\cos 2\theta & 0 \\ 0 & 2\cos 2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we get

$$\cos 2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

35. (c) : Clearly $A + B \neq C$. Hence, Statement-II is false.
 However, by a property of determinants,
 $\det C = \det A + \det B$.

36. (d)

37. (a) : $B = \left[-\frac{1}{2} \sqrt{5} \ 2 \ 3 \right]_{1 \times 4}$ is a row matrix. In
 general, $B = [b_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.

38. (d) : For multiplication of two matrices A and B ,
 the number of columns in A should be equal to the
 number of rows in B . Furthermore, for getting the
 elements of the product matrix, we take rows of A and
 columns of B , multiply them elementwise and take the
 sum.

39. (d) : (P) Given, A is a skew-symmetric matrix.

$$\therefore A' = -A$$

$$\text{Also, } A^2 + I = 0 \Rightarrow A^2 = -I$$

$$\Rightarrow A \cdot A = -I \Rightarrow A \cdot A \cdot A' = -I A' = I(-A') = IA = A$$

$$\Rightarrow (A^{-1} A) AA' = A^{-1} A \Rightarrow AA' = I$$

(Q) $(I + A)^n = C_0 I^n + C_1 I^{n-1} A + C_2 I^{n-2} A^2 + \dots + C_n I^{n-n} A^n$.

$$= C_0 I + C_1 A + C_2 A + \dots + C_n A = I + (C_1 + C_2 + \dots + C_n) A$$

$$= I + (2^n - 1) A \quad [\because (C_1 + C_2 + \dots + C_n) = 2^n - 1]$$

$$\Rightarrow \lambda = 2^n - 1$$

(R) We have, $A^2 = A$ and $B = I - A$

$$\text{Now, } AB + BA + I - (I - A)^2$$

$$= AB + BA + I - (I + A^2 - 2A)$$

$$= AB + BA - A + 2A \quad (\because A^2 = A)$$

$$= AB + BA + A = A(I - A) + (I - A)A + A$$

$$= A - A^2 + A - A^2 + A = A - A + A - A + A = A$$

(S) We have, $A' = A, B' = B$

$$\text{Now, } (AB - BA)' = B'A' - A'B' = BA - AB$$

$$\therefore (AB - BA)' - (BA - AB) = 0$$

40. (a) : We have,

$$\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = px^5 + qx^4 + rx^3 + sx^2 + tx + w$$

$$\text{Consider, } \begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix}$$

SAMURAI SUDOKU

ANSWER - SEPTEMBER 2022



7	1	9	6	2	3	8	4	5	2	9	6	3	4	8	5	1	7
8	6	2	5	4	7	1	3	9	5	8	3	6	7	1	2	4	9
5	3	4	1	8	9	7	6	2	1	4	7	9	5	2	3	6	8
6	4	1	7	5	8	2	9	3	4	1	5	7	6	3	9	8	2
9	2	5	4	3	1	6	7	8	3	2	8	1	9	4	7	5	6
3	7	8	9	6	2	5	1	4	7	6	9	2	8	5	1	3	4
4	8	7	2	9	6	3	5	1	2	6	8	9	7	4	5	3	6
2	5	6	3	1	4	9	8	7	4	1	3	6	5	2	8	1	9
1	9	3	8	7	5	4	2	6	9	5	7	8	3	1	4	2	7
									2	1	8	3	7	6	5	4	9
									7	4	3	8	9	5	1	2	6
									6	9	5	1	2	4	7	8	3
5	9	2	3	7	8	1	6	4	7	8	2	3	9	5	2	8	4
1	4	7	5	2	6	8	3	9	5	4	1	2	6	7	3	9	1
3	6	8	4	1	9	5	7	2	6	3	9	4	1	8	6	5	7
4	8	5	1	3	7	2	9	6	8	5	1	9	2	6	7	3	4
9	2	3	6	5	4	7	8	1	9	2	3	4	7	5	6	8	1
6	7	1	9	8	2	4	5	3	7	4	6	1	3	8	2	5	9
8	3	6	7	4	1	9	2	5	5	3	9	8	1	2	4	7	6
2	1	9	8	6	5	3	4	7	6	7	2	5	4	9	3	1	8
7	5	4	2	9	3	6	1	8	1	8	4	7	6	3	5	9	2

$$= \begin{vmatrix} 1+2x+x^2 & x & x^2 \\ 1+2x+x^2 & 1+x & x^2 \\ 1+2x+x^2 & x & 1+x \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= (1+2x+x^2) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1+x & x^2 \\ 1 & x & 1+x \end{vmatrix}$$

[Taking out $(1+2x+x^2)$ common from C_1]

$$= (1+2x+x^2) \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & x^2-1-x \\ 1 & x & 1+x \end{vmatrix}$$

[Applying $R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$]

$$\begin{aligned} &= (1+2x+x^2)(1+x-x^2) \\ &= 1+x-x^2+2x+2x^2-2x^3+x^2+x^3-x^4 \\ &= 0 \cdot x^5 - 1 \cdot x^4 - 1 \cdot x^3 + 2 \cdot x^2 + 3x + 1 \end{aligned}$$

Comparing with $px^5 + qx^4 + rx^3 + sx^2 + tx + w$, we get

$$p = 0, q = -1, r = -1, s = 2, t = 3, w = 1$$

$$(P) \quad w = 1 \qquad (Q) \quad t = 3$$

$$(R) \quad p + r + w = 0 - 1 + 1 = 0$$

$$(S) \quad q + r = -1 - 1 = -2$$

41. (c) : Let ₹ A , ₹ B and ₹ C be the cost incurred by the organisation for villages X , Y and Z respectively. Then A , B , C will be given by the following matrix equation.

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

42. (c)

43. (b)

44. (c) : Total number of toilets that can be expected in each village is given by the following matrix

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix} = \begin{bmatrix} 8 + 12 + 20 \\ 6 + 10 + 15 \\ 10 + 16 + 30 \end{bmatrix} = \begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

45. (d)

(46-50) : Let the cost of a polythene bag = ₹ x ,
the cost of a handmade bag = ₹ y

and the cost of a newspaper bag = ₹ z

According to the question,

$$20x + 30y + 40z = 250, 30x + 40y + 20z = 270,$$

$$40x + 20y + 30z = 200$$

This system can be written as $AX = B$, where

$$A = \begin{bmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{vmatrix}$$

$$= 20(1200 - 400) - 30(900 - 800) + 40(600 - 1600)$$

$$= 20(800) - 30(100) + 40(-1000)$$

$$= 16000 - 3000 - 40000 = -27000 \neq 0$$

So, A^{-1} exists and system has a solution given by $X = A^{-1}B$.

$$\text{Now, } \text{adj}(A) = \begin{bmatrix} 800 & -100 & -1000 \\ -100 & -1000 & 800 \\ -1000 & 800 & -100 \end{bmatrix}'$$

$$= \begin{bmatrix} 800 & -100 & -1000 \\ -100 & -1000 & 800 \\ -1000 & 800 & -100 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{-27000} \begin{bmatrix} 800 & -100 & -1000 \\ -100 & -1000 & 800 \\ -1000 & 800 & -100 \end{bmatrix}$$

$$\text{Now } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27000} \begin{bmatrix} -800 & 100 & 1000 \\ 100 & 1000 & -800 \\ 1000 & -800 & 100 \end{bmatrix} \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$$

$$= \frac{1}{27000} \begin{bmatrix} 27000 \\ 135000 \\ 54000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} \Rightarrow x = 1, y = 5, z = 2$$

Hence, cost of a polythene bag, a handmade bag and a newspaper envelope are ₹ 1, ₹ 5 and ₹ 2 respectively.

46. (a)

47. (d)

48. (b)

49. (b) : Vijay investing most of the money on hand-made bags.

50. (a) : Salim investing less amount of money on polythene bags.

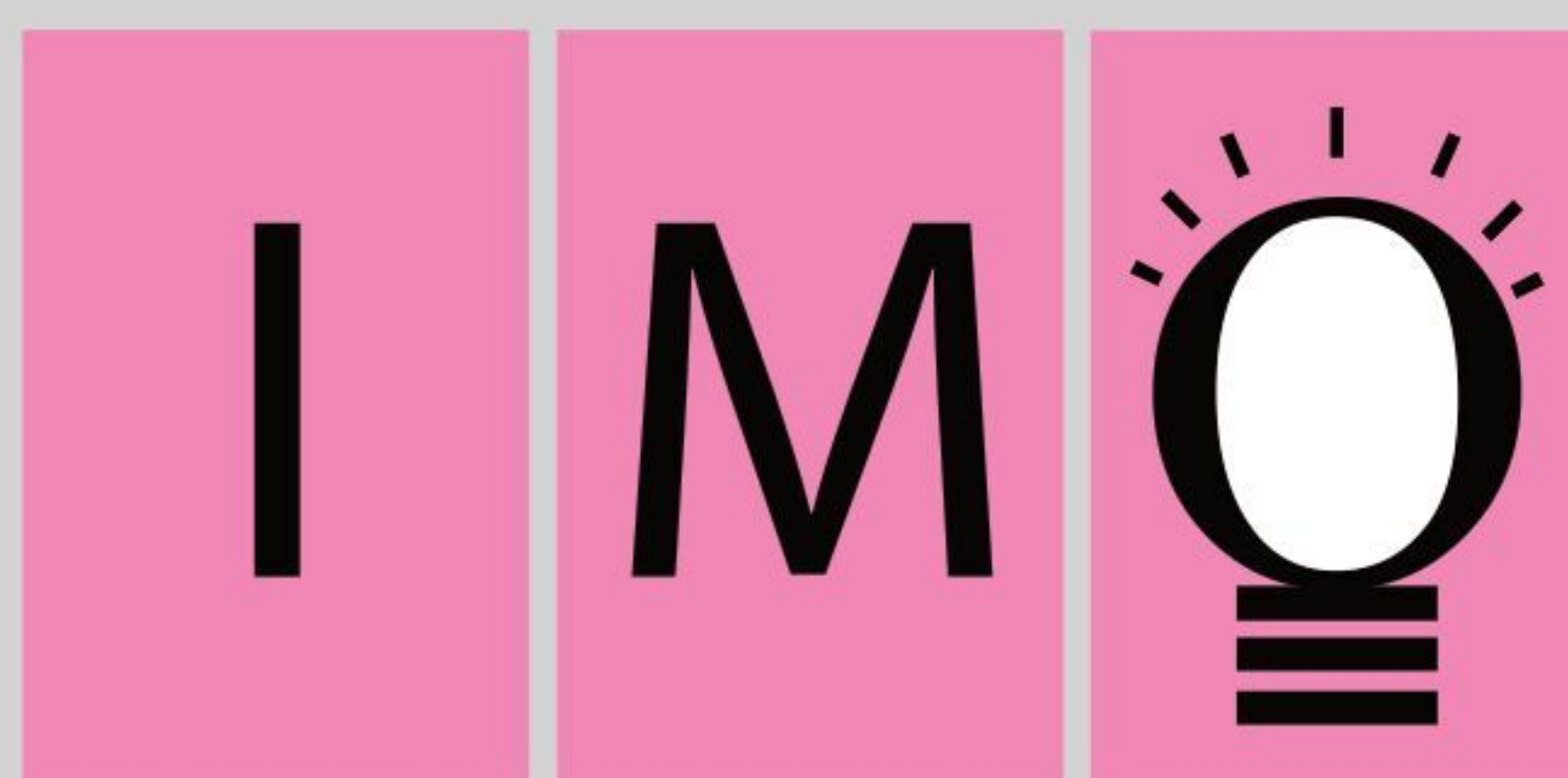




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CLASS XI

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Sets, Relations and Functions, Principle of Mathematical Induction, Logarithms, Complex Numbers & Quadratic Equations, Linear Inequations, Sequences and Series, Trigonometry, Straight Lines, Conic Sections, Permutations and Combinations, Binomial Theorem, Statistics, Mathematical Reasoning, Limits and Derivatives, Probability, Introduction to 3-D Geometry.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Mathematical Reasoning, Limits, Continuity and Differentiability, Probability, Descriptive Statistics, Basics of Financial Mathematics, Straight Lines, Circles.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Limits and Derivatives, Straight Lines, Circles, Probability.

Practice Questions

- If the complex number $z = x + iy$ satisfies the condition $|z + 1| = 1$, then z lies on
 - x -axis
 - circle with centre $(1, 0)$ and radius 1
 - circle with centre $(-1, 0)$ and radius 1
 - y -axis
- Find the equation of the ellipse whose foci are $(2, 3)$, $(-2, 3)$ and whose semi-minor axis is $\sqrt{5}$.
 - $5x^2 + 9y^2 + 54y - 36 = 0$
 - $9x^2 + 5y^2 - 54x + 36 = 0$
 - $5x^2 + 9y^2 - 54y + 36 = 0$
 - $9x^2 + 5y^2 + 54x - 36 = 0$
- The value of $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is equal to
 - 11
 - 12
 - 13
 - 14
- For any three sets A , B and C the set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to
 - $B \cap C'$
 - $B' \cap C'$
 - $B \cap C$
 - $A \cap B \cap C$
- The probability that a leap year selected at random contains either 53 Sundays or 53 Mondays, is
 - $2/7$
 - $4/7$
 - $3/7$
 - $1/7$
- If a point C with y coordinate 2, lies on the line joining the points $A(-1, -4, 5)$ and $B(4, 6, -5)$, then find the coordinates of C .
 - $(2, 2, -1)$
 - $(2, 1, 2)$
 - $(2, 0, 0)$
 - $(0, 2, 4)$
- The sum of the first n terms of the arithmetic progression is equal to half the sum of the next n terms of the same progression. Find the ratio of the sum of the first $3n$ terms of the progression to the sum of its first n terms.
 - $5 : 2$
 - $6 : 1$
 - $7 : 3$
 - $8 : 5$
- The equation $\sin^4 x - 2 \cos^2 x + a^2 = 0$ is solvable for $x \in R$, if
 - $-\sqrt{2} \leq a \leq \sqrt{2}$
 - $-\sqrt{3} \leq a \leq \sqrt{3}$
 - $-1 \leq a \leq 1$
 - None of these
- The value of the expansion $(\sqrt{3} + 1)^5 + (\sqrt{3} - 1)^5$ equals
 - 88
 - 40
 - $88\sqrt{3}$
 - $40\sqrt{3}$
- The range of the function $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$, where $x \in R$ is
 - $(-\infty, 3]$
 - $(-\infty, \infty)$
 - $[3, \infty)$
 - $\left[\frac{1}{3}, 3\right]$
- If $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$, then find $\tan 2x$.
 - 1
 - 0
 - 1
 - $\sqrt{3}$

12. If $|z^2 - 1| = |z|^2 + 1$, then z lies on
 A. imaginary axis B. real axis
 C. origin D. None of these
13. How many five-digit numbers divisible by 6 can be formed by using 0, 1, 2, 3, 4, 5 without repetition?
 A. 72 B. 96
 C. 108 D. None of these
14. For hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains constant with change in ' α '?
 A. Abscissae of vertices B. Abscissae of foci
 C. Eccentricity D. Directrix
15. If a parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then find the length of the diagonal.
 A. 7 units B. 5 units C. 8 units D. 3 units

ACHIEVERS SECTION

16. Match the following.

Column-I

P. $\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x} =$

Q. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x} =$

R. $\lim_{x \rightarrow 0} \frac{\tan 3x - 2x}{3x - \sin^2 x} =$

S. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} =$

- A. $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 1$
 B. $P \rightarrow 1, Q \rightarrow 2, R \rightarrow 4, S \rightarrow 3$
 C. $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 4, S \rightarrow 1$
 D. $P \rightarrow 1, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 3$

Column-II

1. $1/2$

2. $1/3$

3. -1

4. 2

17. Which of the following statements is incorrect?
 A. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.
 B. If the equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
 Then, no portion of curve lies between the lines $x = +a$ and $x = -a$.

- C. The foci of an ellipse always lie on major axis.
 D. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

18. Read the statements carefully and select the correct option.

Statement-I : Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is 4 units.

Statement-II : The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the end of its latus rectum is 18 sq. units.

- A. Both Statement-I and Statement-II are true.
 B. Statement-I is true but Statement-II is false.
 C. Statement-I is false but Statement-II is true.
 D. Both Statement-I and Statement-II are false.

19. Read the statements carefully and state 'T' for true and F for false.

- (i) A box contains 100 bulbs, 20 of which are defective 10 bulbs are selected for inspection.

The probability that at least one bulb is defective is $1 - \frac{{}^{80}C_{10}}{{}^{100}C_{10}}$.

- (ii) In a single throw of three dice, the probability of getting the same number on all the three dice is $1/12$.

- (iii) If an integer is chosen at random from first 200 positive integers, then the probability that the chosen integer is divisible by 6 or 8 is $1/4$.

- | | (i) | (ii) | (iii) |
|----|-----|------|-------|
| A. | F | F | T |
| B. | T | T | F |
| C. | T | F | T |
| D. | T | F | F |

20. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

The mean diameter and standard deviation of the circles respectively are

- A. 44.5, 5.55 B. 43.5, 5.25
 C. 43.5, 5.55 D. 44.5, 5.25

Darken your choice with HB Pencil

1. (A) (B) (C) (D)	5. (A) (B) (C) (D)	9. (A) (B) (C) (D)	13. (A) (B) (C) (D)	17. (A) (B) (C) (D)
2. (A) (B) (C) (D)	6. (A) (B) (C) (D)	10. (A) (B) (C) (D)	14. (A) (B) (C) (D)	18. (A) (B) (C) (D)
3. (A) (B) (C) (D)	7. (A) (B) (C) (D)	11. (A) (B) (C) (D)	15. (A) (B) (C) (D)	19. (A) (B) (C) (D)
4. (A) (B) (C) (D)	8. (A) (B) (C) (D)	12. (A) (B) (C) (D)	16. (A) (B) (C) (D)	20. (A) (B) (C) (D)

SOLUTIONS

1. (C) : $|z + 1| = 1 \Rightarrow |(x + 1) + iy| = 1$
 $\Rightarrow (x + 1)^2 + y^2 = 1$, which is a circle with centre $(-1, 0)$ and radius 1.

2. (C) : We have, $b = \sqrt{5}$
 $SS' = 4$ and $2ae = 4 \Rightarrow ae = 2$
 $\therefore b^2 = a^2(1 - e^2) \Rightarrow 5 = a^2 - 4 \Rightarrow a^2 = 9$
 let $P(\alpha, \beta)$ be any point on the ellipse
 $\therefore SP + S'P = 2a$
 $\Rightarrow \sqrt{(\alpha - 2)^2 + (\beta - 3)^2} + \sqrt{(\alpha + 2)^2 + (\beta - 3)^2} = 6$
 $\Rightarrow (\alpha - 2)^2 + (\beta - 3)^2$
 $= 36 + (\alpha + 2)^2 + (\beta - 3)^2 - 12\sqrt{(\alpha + 2)^2 + (\beta - 3)^2}$
 $\Rightarrow -8\alpha - 36 = -12\sqrt{(\alpha + 2)^2 + (\beta - 3)^2}$
 $\Rightarrow 2\alpha + 9 = 3\sqrt{(\alpha + 2)^2 + (\beta - 3)^2}$
 Squaring both sides, we get
 $(2\alpha + 9)^2 = 9\{(\alpha + 2)^2 + (\beta - 3)^2\}$
 $\Rightarrow 4\alpha^2 + 81 + 36\alpha = 9\alpha^2 + 9\beta^2 + 36\alpha - 54\beta + 117$
 $\Rightarrow 5\alpha^2 + 9\beta^2 - 54\beta + 36 = 0$
 Required equation of ellipse is
 $5x^2 + 9y^2 - 54y + 36 = 0$

3. (C) : We have, $3(\sin x - \cos x)^4$
 $+ 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$
 $= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4[(\sin^2 x + \cos^2 x)^3$
 $- 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)]$
 $= 3(1 - 2\sin 2x + \sin^2 2x) + 6 + 6\sin 2x$
 $+ 4\left[1 - \frac{3}{4}\sin^2 2x\right]$
 $= 13 + 3\sin^2 2x - 3\sin^2 2x = 13.$

4. (A) : Since, $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$
 $= (A \cup (B \cup C)) \cap (A' \cup (B \cup C)) \cap C'$
 $= (A \cap A') \cup (B \cup C) \cap C'$
 $= \phi \cup (B \cup C) \cap C' = B \cap C' \cup \phi = B \cap C'$

5. (C) : Total number of days in a leap year is 366 containing 52 weeks and 2 days.

These two days can be

$S = \{(\text{Sun, Mon}), (\text{Mon, Tues}), (\text{Tues, Wed}), (\text{Wed, Thus}), (\text{Thus, Fri}), (\text{Fri, Sat}), (\text{Sat, Sun})\}$

Therefore $n(S) = 7$

Let A be event of getting 53 Sundays

Therefore $A = \{(\text{Sun, Mon}), (\text{Sat, Sun})\}$

For 53 Sundays, Probability $P(A) = 2/7$

Similarly, let B be event of getting 53 Mondays

So, $B = \{(\text{Mon, Tues}), (\text{Sun, Mon})\}$

Therefore, for 53 Mondays, $P(B) = \frac{2}{7}$

Here, it include one possibility where Sunday and Monday occur simultaneously.

i.e., $A \cap B = \{\text{Sun, Mon}\}$

$\therefore P(A \cap B) = \frac{1}{7}$

Hence, required probability is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}.$$

6. (A) : Line joining $A(-1, -4, 5)$ and $B(4, 6, -5)$ is

$$\frac{x - 4}{-1 - 4} = \frac{y - 6}{-4 - 6} = \frac{z - (-5)}{5 - (-5)} \Rightarrow \frac{x - 4}{-5} = \frac{y - 6}{-10} = \frac{z + 5}{10}$$

$$\Rightarrow \frac{x - 4}{1} = \frac{y - 6}{2} = \frac{z + 5}{-2} = r$$

$\therefore$ General coordinates of a point on this line is given by,

$$(x, y, z) = (r + 4, 2r + 6, -2r - 5)$$

Given, y coordinate = 2

$$\therefore 2r + 6 = 2$$

$$\Rightarrow r = -2$$

$$\therefore (x, y, z) = (2, 2, -1)$$

7. (B) : Let the first term of the A.P be a and common difference be d .

Given, the sum of first n terms of the A.P is equal to half the sum of the next n term of the same progression. This suggests that the sum of the first $2n$ terms is thrice the sum of the first n terms.

$$\therefore \frac{S_{2n}}{S_n} = 3 \Rightarrow \frac{\frac{2n}{2}[2a + (2n - 1)d]}{\frac{n}{2}[2a + (n - 1)d]} = 3$$

$$\Rightarrow 4a + 4nd - 2d = 6a + 3nd - 3d$$

$$\Rightarrow 2a = nd + d \Rightarrow 2a - d = nd \quad \dots(i)$$

$$\text{Now, } \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a + (3n - 1)d]}{\frac{n}{2}[2a + (n - 1)d]}$$

$$= \frac{6a + 9nd - 3d}{2a + nd - d} = 3 + \frac{6nd}{2a + nd - d} = 3 + \frac{6nd}{(2a - d) + nd}$$

$$= 3 + \frac{6nd}{nd + nd} \quad [\text{Using (i)}]$$

$$= 3 + \frac{6nd}{2nd} = 3 + 3 = 6. \text{ Hence, } S_{3n} : S_n = 6 : 1$$

8. (A) : We have, $\sin^4 x - 2\cos^2 x + a^2 = 0$

Let $\sin^2 x = y$, then the given equation becomes

$$y^2 - 2(1 - y) + a^2 = 0$$

$$\Rightarrow y^2 + 2y + a^2 - 2 = 0 \Rightarrow y = -1 \pm \sqrt{3 - a^2}$$

For y to be real, Discriminant ≥ 0

$$\Rightarrow (2)^2 - 4 \times 1 \times (a^2 - 2) \geq 0$$

$$\Rightarrow 3 - a^2 \geq 0 \Rightarrow a^2 \leq 3$$

...(i)

Now, $y = \sin^2 x$

$$\therefore 0 \leq y \leq 1 \Rightarrow 0 \leq -1 \pm \sqrt{3 - a^2} \leq 1$$

$$\Rightarrow 1 \leq \pm \sqrt{3 - a^2} \leq 2 \Rightarrow 1 \leq 3 - a^2 \leq 4$$

$$\Rightarrow -2 \leq -a^2 \leq 1 \Rightarrow a^2 - 2 \leq 0 \text{ and } a^2 + 1 \geq 0 \quad \dots(\text{ii})$$

Using (i) and (ii), we have

$$a^2 \leq 2 \Rightarrow -\sqrt{2} \leq a \leq \sqrt{2}$$

9. (C) : Since, $[(a + b)^5 + (a - b)^5] = 2[a^5 + 10a^3b^2 + 5ab^4]$

$$\begin{aligned} \therefore (\sqrt{3} + 1)^5 + (\sqrt{3} - 1)^5 &= 2[(\sqrt{3})^5 + 10(\sqrt{3})^3 + 5\sqrt{3}] \\ &= 2(\sqrt{3})[(\sqrt{3})^4 + 10(\sqrt{3})^2 + 5] = 88\sqrt{3} \end{aligned}$$

10. (D) : We have, $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$$\Rightarrow yx^2 + yx + y - x^2 + x - 1 = 0$$

$$\Rightarrow x^2(y - 1) + x(y + 1) + (y - 1) = 0$$

$$\therefore x = \frac{-(y + 1) \pm \sqrt{(y + 1)^2 - 4(y - 1)^2}}{2(y - 1)}$$

$$= \frac{-(y + 1) \pm \sqrt{-3y^2 + 10y - 3}}{2(y - 1)}$$

If $y = 1$, then original equation gives $x = 0$.

$$\text{Also, } -3y^2 + 10y - 3 \geq 0 \Rightarrow 3y^2 - 10y + 3 \leq 0$$

$$\Rightarrow 3y^2 - 9y - y + 3 \leq 0 \Rightarrow (3y - 1)(y - 3) \leq 0$$

$$\Rightarrow y \in \left[\frac{1}{3}, 3\right] \therefore \text{Range is } \left[\frac{1}{3}, 3\right].$$

11. (C) : Given $\sin x - 3\sin 2x + \sin 3x = \cos x$

$$-3\cos 2x + \cos 3x$$

$$\Rightarrow (\sin 3x + \sin x) - 3\sin 2x = (\cos 3x + \cos x) - 3\cos 2x$$

$$\Rightarrow 2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cos x - 3\cos 2x$$

$$\Rightarrow \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$$

$$\Rightarrow \sin 2x = \cos 2x \quad \left(\because \cos x \neq \frac{3}{2}\right)$$

$$\Rightarrow \tan 2x = 1$$

12. (A) : Let $z = x + iy$ Then, $|z^2 - 1| = |z|^2 + 1$

$$\Rightarrow |x^2 - y^2 - 1 + i2xy| = |x + iy|^2 + 1$$

$$\Rightarrow (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2 \Rightarrow 4x^2 = 0 \Rightarrow x = 0$$

Hence z lies on y -axis or imaginary axis.

13. (C) : Here we need to use divisibility rule for both 2 (last digit is even) and 3 (sum of digit divisible by 3).

First we select the 5 digits whose sum is divisible by 3

Sum of all given digits = 15, which is multiple of 3.

We need to leave out a single digit such that the sum remains a multiple of 3.

We have, the following two cases:

Case I: 1, 2, 3, 4, 5 are chosen.

For the number to be divisible by 2, last digit is 2 or 4, i.e., last digit can be selected in 2 ways and the other digits can arrange in 4! ways.

$$\text{Thus number of ways} = 4! \times 2 = 24 \times 2 = 48$$

**Mentor
Advice**

5 Steps to Excel in Olympiads

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- 1 Make an Appropriate Use of Time Tool :** Get to know your syllabus, make a timetable for various subjects, giving more time to the subjects you are weak in.
- 2 Keep Revising at Regular Intervals :** Revise all the topics at regular intervals, as it is imperative for any exam preparation. It helps in remembering the facts, figures, topics and methodologies already covered by you and will surely increase your confidence.
- 3 Practice Previous Years' Papers :** Practice as many previous years' question papers as possible, as it gives a fair idea about the level of difficulty and pattern of questions.
- 4 Study with Good Resource Books :** A good resource book is like a good 'Mentor'. Prepare for Olympiads with right guidance as well as with the best trusted resources.
- 5 Stay Motivated :** Last but not the least, always stay positive and focused. Believe in Yourself!

Case II: 0, 1, 2, 4, 5 are chosen.

(a) If last digit is 0, then number of ways = $4! = 24$

(b) If last digit is 2, then other digits may be filled in (from left to Right) = $3 \times 3 \times 2 \times 1 = 18$ ways

(c) If last digit is 4, then the other digits may be filled in (from left to Right) = $3 \times 3 \times 2 \times 1 = 18$ ways

So, required answer = $48 + 24 + 18 + 18 = 108$ ways

14. (B) : $b^2 = a^2 (e^2 - 1)$

Here $a = \cos \alpha$, $b = \sin \alpha$

So, $\sin^2 \alpha = \cos^2 \alpha (e^2 - 1)$

$\Rightarrow \tan^2 \alpha = e^2 - 1 \Rightarrow e^2 = 1 + \tan^2 \alpha = \sec^2 \alpha$

Vertices $\equiv (\pm \cos \alpha, 0)$

Coordinators of foci are $(\pm ae, 0) \equiv (\pm 1, 0)$

Abscissae of Foci is independent of α .

15. (A) : Length of edges of the parallelopiped are 5 – 2, 9 – 3, 7 – 5 i.e., 3, 6, 2.

$\therefore$ Length of its diagonal = $\sqrt{3^2 + 6^2 + 2^2} = 7$ units.

16. (A) : (P) $\lim_{x \rightarrow 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x}$

$$= \lim_{x \rightarrow 0} \frac{\frac{\cos 2x}{\sin 2x} - \frac{1}{\sin 2x}}{x} = \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{x \sin 2x}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin^2 x}{x \sin 2x} = \lim_{x \rightarrow 0} -2 \left(\frac{\sin x}{x} \right) \left(\frac{\sin x}{\sin 2x} \right) = -1$$

(Q) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x}$

$$= \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x} \times 2 \left(\frac{x}{\sin x} \right) = 2$$

(R) $\lim_{x \rightarrow 0} \frac{\tan 3x - 2x}{3x - \sin^2 x} = \lim_{x \rightarrow 0} \frac{2x \left(\frac{\tan 3x}{2x} - 1 \right)}{\sin x \left(\frac{3x}{\sin x} - \sin x \right)}$

$$= \lim_{x \rightarrow 0} 2 \left(\frac{x}{\sin x} \right) \left(\frac{\frac{3}{2} - 1}{3 - 0} \right) = 1/3$$

(S) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{2 \cos^2 x}{4 \left(\frac{\pi}{2} - x \right)^2}$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{2} \frac{\sin^2 \left(\frac{\pi}{2} - x \right)}{\left(\frac{\pi}{2} - x \right)^2} = \frac{1}{2}$$

17. (D) : Given equation of the circle is $x^2 + y^2 + 6x + 2y = 0$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

Centre = $(-g, -f) = (-3, -1)$

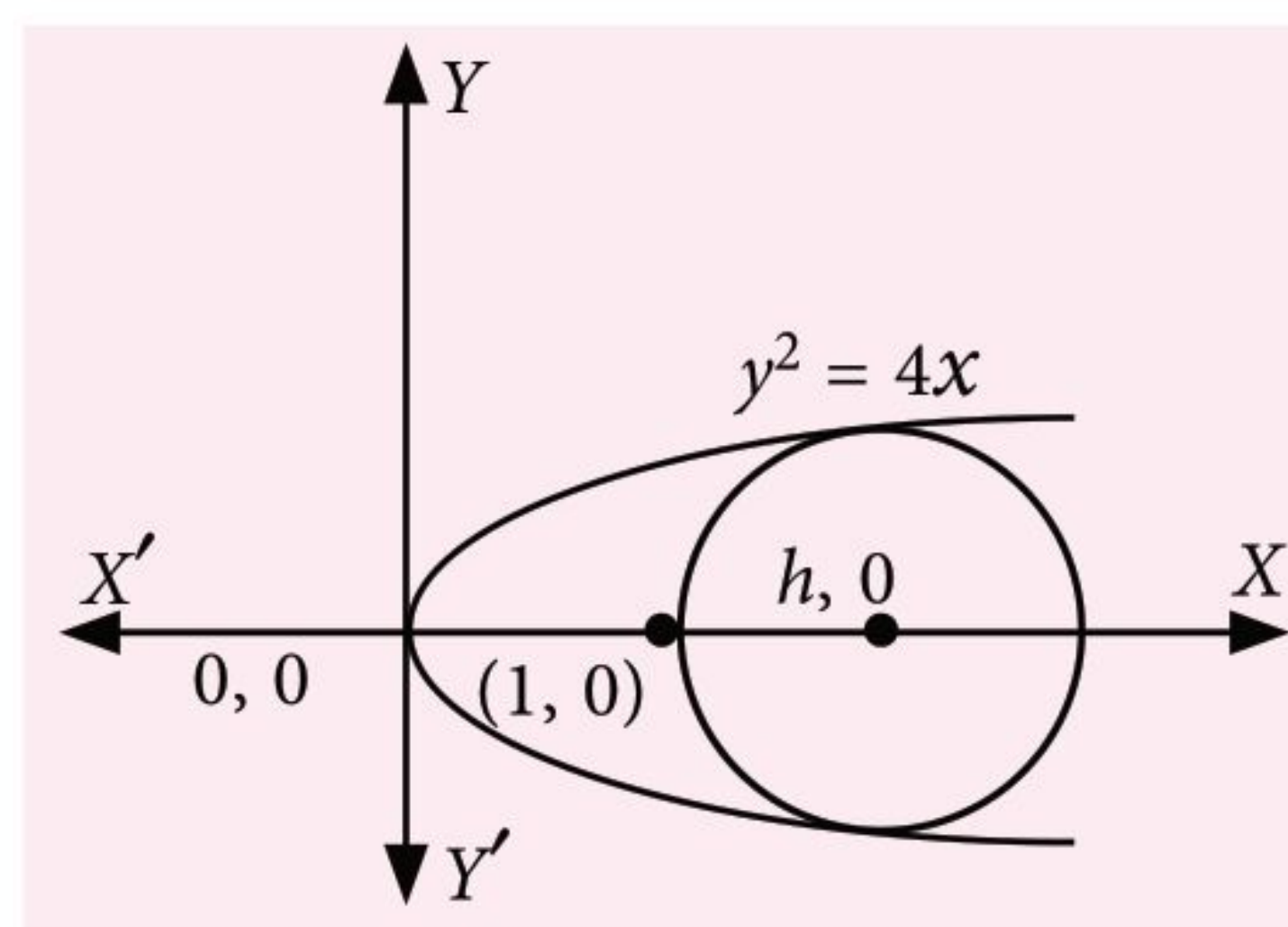
Now, given equation of line is $x + 3y = 0$

$\Rightarrow -3 - 3 = -6 \neq 0$

So, centre does not lie on the given line.

$\therefore$ Given line can not be the diameter of the circle.

18. (A) : (I) let $(h, 0)$ be the centre of circle, then, $h = 1 + r$



Equation of circle is $(x - h)^2 + y^2 = r^2$

$\Rightarrow (x - 1 - r)^2 + y^2 = r^2$

$\Rightarrow x^2 + r^2 + 1 - 2xr + 2r - 2x + y^2 = r^2$

$\Rightarrow x^2 + y^2 - 2xr + 2r - 2x + 1 = 0$

Now, $y^2 = 4x$

[given]

$\Rightarrow x^2 + 4x - 2xr + 2r - 2x + 1 = 0$

$\Rightarrow x^2 + x(2 - 2r) + 2r + 1 = 0$

Since, the circle intersect the parabola.

$\therefore (2 - 2r)^2 = 4 \cdot 1 \cdot (2r + 1)$

$\Rightarrow 4 + 4r^2 - 8r - 8r - 4 = 0$

$\Rightarrow 4r^2 - 16r = 0 \Rightarrow r = 0, 4$

$r = 0$ is not possible.

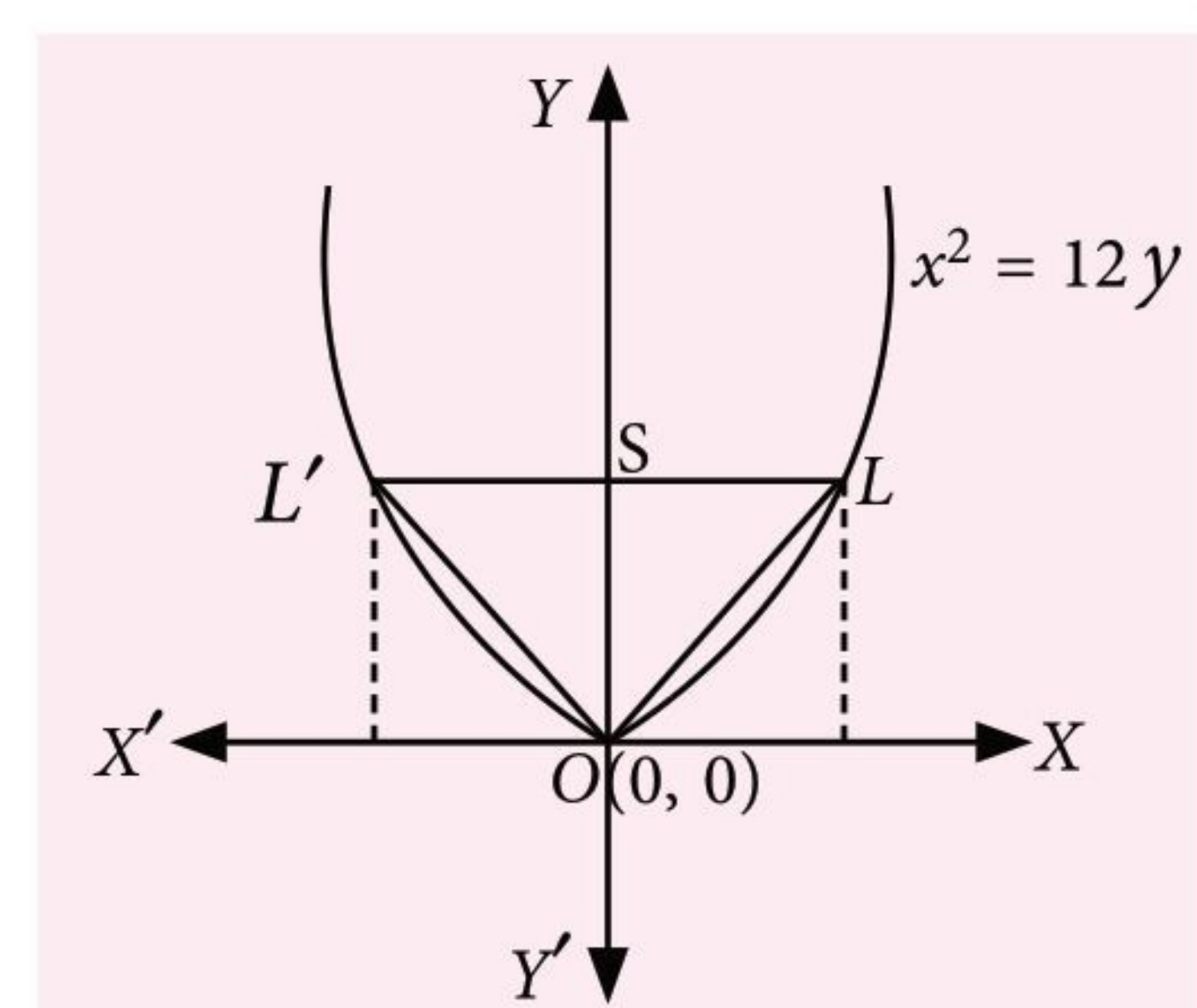
Hence, radius of circle = 4 units

(II) Given equation of parabola is $x^2 = 12y$

$\Rightarrow x^2 = 4 \cdot 3 \cdot y$ [$\because x^2 = 4 \cdot a \cdot y$]

$\therefore$ Focus of parabola is S (0, 3).

End points of later tectum are L (2a, a) and L'(-2a, a) i.e., L (6, 3) and L'(-6, 3).



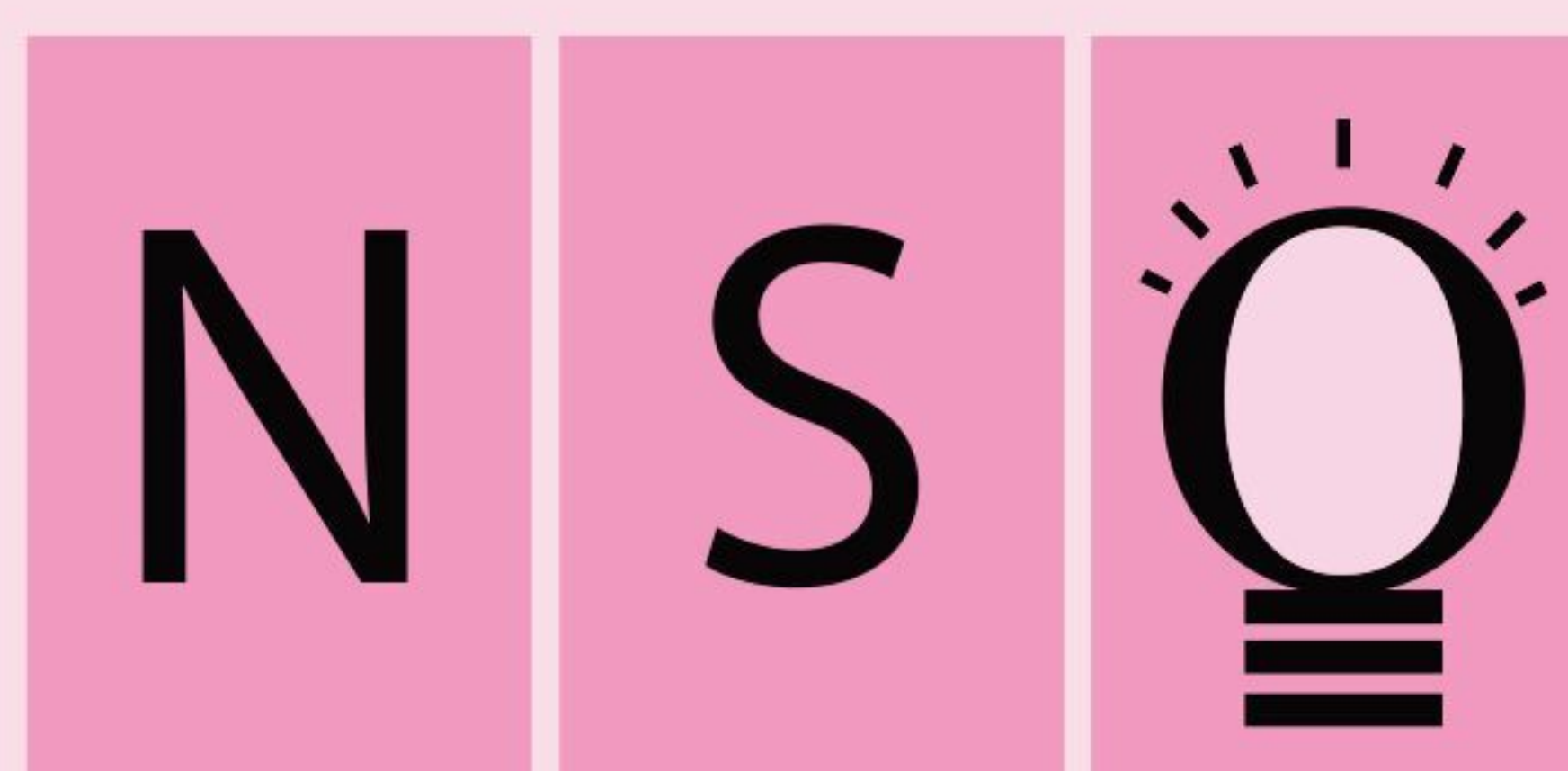
Length of latus rectum, $LL' = 4a = 12$



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$$\therefore \text{Area of } \Delta OLL' = \frac{1}{2} \times 12 \times 3 = 18 \text{ sq. units}$$

19. (C) : (i) Let E be the event that at least one bulb is defective.

$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, where 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are number of defective bulbs

Here, a box containing 100 bulbs out, of which 20 are defective.

$$\therefore \text{Number of good bulbs} = 100 - 20 = 80$$

Now, 10 bulbs are selected for inspection.

$$\therefore \text{Number of elementary events in sample space } n(S) = {}^{100}C_{10}$$

Therefore, probability for at least one defective bulb = 1 - Probability (all 10 bulbs are non defective)

$$= 1 - \frac{{}^{80}C_{10}}{{}^{100}C_{10}}$$

(ii) Total number of outcomes = $6^3 = 216$

Number of favourable outcomes in throwing three dice = $\{(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)\}$ i.e., = 6.

Probability of getting the same number on all the three

$$\text{dice} = \frac{6}{216} = \frac{1}{36}$$

(iii) Let S be sample space, then $n(S) = 200$

Let A be the event that the number selected is divisible by 6 and B be the event that the number selected is divisible by 8.

Then $A = \{6, 12, 18, 24, \dots, 198\}$

$B = \{8, 16, 24, 32, \dots, 200\}$

and $(A \cap B) = \{24, 48, 72, \dots, 192\}$

$$\text{We have, } n(A) = \frac{198}{6} = 33$$

$$n(B) = \frac{200}{8} = 25$$

$$n(A \cap B) = \frac{192}{24} = 8 \quad [\because \text{LCM of 6 and 8} = 24]$$

$$\therefore P(A) = \frac{33}{200}, P(B) = \frac{25}{200} \text{ and } P(A \cap B) = \frac{8}{200}$$

Now, required probability = $P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{1}{4}$$

20. (C) : Since, the given frequency distribution is not continuous, so to make it continuous, we will subtract

0.5 from lower limit and add 0.5 to the upper limit of each class.

Class	f_i	x_i	$\mu_i = \frac{x_i - A}{h}$	μ_i^2	$f_i \mu_i$	$f_i \mu_i^2$
32.5–36.5	15	34.5	-2	4	-30	60
36.5–40.5	17	38.5	-1	1	-17	17
40.5–44.5	21	42.5	0	0	0	0
44.5–48.5	22	46.5	1	1	22	22
48.5–52.5	25	50.5	2	4	50	100
Total	100				25	199

Here, $N = 100$, $h = 4$

let assumed mean, $A = 42.5$

$$\text{Mean } (\bar{x}) = \frac{A + \sum_{i=1}^5 f_i \mu_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$$

$$\text{Variance } (\sigma^2) = \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i \mu_i^2 - \left(\sum_{i=1}^5 f_i \mu_i \right)^2 \right]$$

$$= \frac{16}{10000} [100 \times 199 - (25)^2] = 30.84$$

$$\therefore \text{Standard deviation } (\sigma) = \sqrt{30.84} = 5.55$$

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CLASS XII

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Relations and Functions, Inverse Trigonometric Functions, Matrices and Determinants, Continuity and Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Vector Algebra, Three Dimensional Geometry, Probability, Linear Programming.

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Solutions of Simultaneous Linear Equations, Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Probability, Inferential Statistics, Index numbers, Time-based data, Financial Mathematics, Linear Programming.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Linear Programming, Probability.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

Practice Questions

- The solution curve of $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$, $y(-1) = 1$ is a/an
 A. Straight line B. Parabola
 C. Circle D. Ellipse.
- The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is
 A. $\frac{22}{7} - \pi$ B. $\frac{2}{105}$
 C. 1 D. $\frac{71}{15} - \frac{3\pi}{2}$
- The value of $\Delta = \begin{vmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) & \cos 2\beta \\ \sin \alpha & \cos \alpha & \sin \beta \\ -\cos \alpha & \sin \alpha & \cos \beta \end{vmatrix}$ is independent of
 A. α B. β
 C. α, β D. none of these
- Let p, q , and r be distinct positive numbers. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent, if
 A. $p + q - r = 0$
 B. $p^2 + q^2 + r^2 = -pq + qr + rp$
 C. $p^3 + q^3 + r^3 = 3pqr$
 D. None of these
- If integrating factor of $x(1 - x^2)dy + (2x^2y - y - ax^3)dx = 0$ is $e^{\int Pdx}$, then P is equal to
 A. $\frac{2x^2 - ax^3}{x(1 - x^2)}$ B. $2x^3 - 1$
 C. $\frac{2x^2 - 1}{ax^3}$ D. $\frac{2x^2 - 1}{x(1 - x^2)}$
- The area bounded by the curve $y^2 = 9x$ and the lines $x = 1, x = 4$ and $y = 0$ in the first quadrant is
 A. 7 sq. units B. 14 sq. units
 C. 28 sq. units D. 14/3 sq. units
- If A and B are two non singular matrices and both are symmetric and commute each other, then
 A. Both $A^{-1}B$ and $A^{-1}B^{-1}$ are symmetric.
 B. $A^{-1}B$ is symmetric but $A^{-1}B^{-1}$ is not symmetric.
 C. $A^{-1}B^{-1}$ is symmetric but $A^{-1}B$ is not symmetric.
 D. Neither $A^{-1}B$ nor $A^{-1}B^{-1}$ are symmetric.
- Let A be a $n \times n$ matrix with $|A| = 4$. B is the adjoint matrix of the matrix $2A$ such that $|B| = 1024$. What is the value of n ?
 A. 3 B. 4
 C. 5 D. More than 5

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CONCEPT MAP

SEQUENCE AND SERIES

Class XI

Class XII

CONTINUITY

CONCEPT MAP

Sequence

A sequence is a function from natural number N (domain) to real numbers (co-domain)

Series

If $a_1, a_2, \dots, a_n$ is a sequence, then the expansion $a_1 + a_2 + \dots + a_n$ is called the series. The series is finite or infinite depending if the sequence is finite or infinite.

Progression

If the terms of a sequence are written under specific conditions, then the sequence is called progression.

Types of Progression

Sum of n terms of Special Series

- Sum of first n natural numbers, $\sum_{n=1}^n n = \frac{n(n+1)}{2}$
- Sum of squares of first n natural numbers, $\sum_{n=1}^n n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first n natural numbers, $\sum_{n=1}^n n^3 = \frac{n^2(n+1)^2}{4} = (\sum_{n=1}^n n)^2$
- $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$
- $1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Arithmetic Progression (A.P.)

A sequence whose terms increases or decreases by a fixed number.

n^{th} term : $T_n = a + (n-1)d$, where d (common difference)

$= T_n - T_{n-1}$, a = first term,
 n^{th} term from end
 $T_n' = l - (n-1)d$, where l = last term

Sum of n terms

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$$

Note

- 3 terms in an A.P. : $a-d, a, a+d$
- 4 terms in an A.P. : $a-3d, a-d, a+d, a+3d$
- 5 terms in an A.P. : $a-2d, a-d, a, a+d, a+2d$

Basic Properties

- If $a_1, a_2, a_3, \dots, a_n$ are in A.P. with common difference d , then
 - $a_1 \pm x, a_2 \pm x, \dots, a_n \pm x$ are also in A.P., where x is a constant.
 - $ka_1, ka_2, \dots, ka_n$ are also in A.P. with common difference kd .
 - $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \dots, \frac{a_n}{k}$ are also in A.P. with common difference, $\frac{d}{k}, k \neq 0$.

Geometric Progression (G.P.)

A sequence of non-zero numbers for which the ratio of a term to its just preceding term is always constant.

n^{th} term : $T_n = ar^{n-1}$, where r (common ratio) = T_n / T_{n-1} , a = first term

n^{th} term from end : $T_n' = l / r^{n-1}$, l = last term

Sum of n terms

$$S_n = \begin{cases} \frac{a(r^n - 1)}{r - 1}, & r > 1 \\ \frac{a(1 - r^n)}{1 - r}, & r < 1 \\ an, & r = 1 \end{cases}; S_{\infty} = \frac{a}{1-r}, |r| < 1$$

Note

- 3 terms in G.P. : $a/r, a, ar$.
- 4 terms in G.P. : $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
- 5 terms in G.P. : $\frac{a}{r^4}, \frac{a}{r^2}, a, ar, ar^2$.

Basic Properties

- If $a_1, a_2, a_3, \dots, a_n$ are in G.P., with common ratio r , then
 - $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ also in G.P. with common ratio $1/r$.
 - $ka_1, ka_2, ka_3, \dots, ka_n$ or $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \dots, \frac{a_n}{k}$ ($k \neq 0$) also in G.P., with common ratio r .
 - $a_1^n, a_2^n, a_3^n, \dots, a_n^n$ also in G.P. with common ratio r^n .
 - $a_1 a_n = a_2 a_{n-1} = \dots = a_k a_{n-k+1} \forall k = 1, 2, 3, \dots, n-1$

Means

Arithmetic mean (A.M.)

- For two numbers a and b , A.M. is $\frac{a+b}{2}$.

- $A_r = a + r \left(\frac{b-a}{n+1} \right), \forall r = 1, 2, \dots, n$ where $A_1, A_2, \dots, A_n$ are n arithmetic means inserted between two numbers a and b .

Geometric mean (G.M.)

- For two numbers a and b , G.M. is $\sqrt{ab}$.
- $G_k = a \left(\frac{b}{a} \right)^{\frac{k}{n+1}} \forall k = 1, 2, 3, \dots, n$, where $G_1, G_2, \dots, G_n$ are n geometric means inserted between two numbers a and b .

CONTINUITY

A function $f(x)$ is said to be continuous if its graph has no break or hole on it.

Continuity of a Function at a Point

The real valued function $y = f(x)$ is said to be continuous at $x = a$ if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$.

Continuity of a Function in an Interval

- $f(x)$ is said to be continuous in an open interval (a, b) if it is continuous at each point of the interval.
- $f(x)$ is said to be continuous in the closed interval $[a, b]$ if $f(x)$ is continuous in (a, b) , $\lim_{x \rightarrow a^+} f(x) = f(a)$ and $\lim_{x \rightarrow b^-} f(x) = f(b)$.

Continuity Everywhere

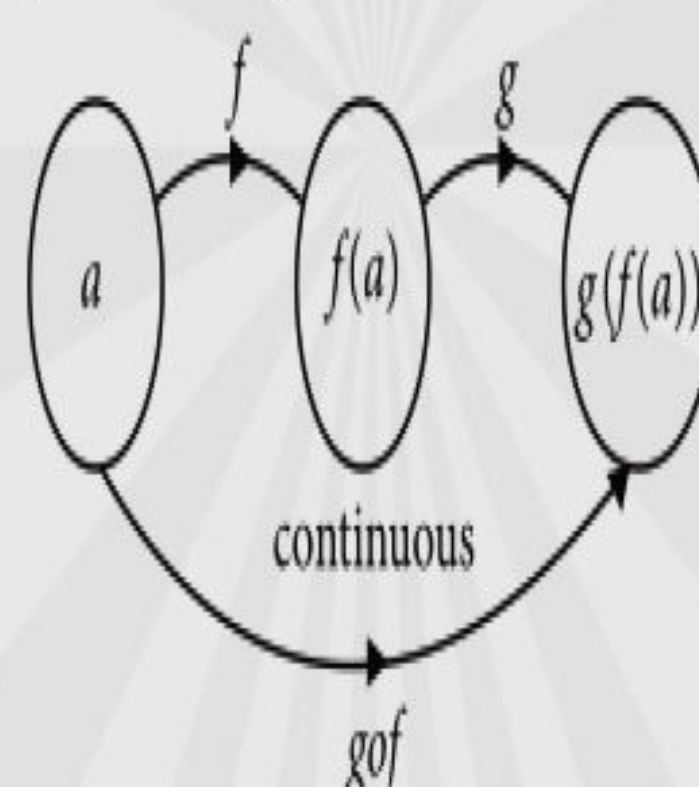
A function is said to be continuous everywhere if it is continuous on the entire real number line $(-\infty, \infty)$.

Algebra of Continuous Functions

- If $f(x)$ is continuous, then $kf(x)$ is continuous for any constant k .
- If $f(x)$ and $g(x)$ are continuous, then $f(x) \pm g(x)$ and $f(x)g(x)$ are continuous in their domain. Also, $\frac{f(x)}{g(x)}$ is continuous at all values of x for which $g(x) \neq 0$.
- $f^n(x), \forall n \in N$ is continuous.

Continuity of Composition of Two Functions

If f, g are continuous functions, then $f \circ g$ and $g \circ f$ are continuous. If f is continuous at a point $x = a$ and g is continuous at $f(a)$, then $g \circ f$ is continuous at $x = a$.



DISCONTINUITY

The function $f(x)$ will be discontinuous at $x = a$ if it is not continuous at that point.

Types of Discontinuity

- Discontinuity of first kind (Jump Discontinuity) :** The function $f(x)$ is said to have discontinuity of first kind or jump discontinuity at $x = a$ if $\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$ both exist finitely but are not equal.
- Discontinuity of second kind (Oscillating Discontinuity) :** The function $f(x)$ is said to have discontinuity of second kind (oscillating discontinuity) at $x = a$ if $\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$ both do not exist finitely or do not approach a definite value at $x = a$.
- Removable Discontinuity :** The function $f(x)$ is said to have removable discontinuity at $x = a$ if $\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$ both exist and are equal but not equal to $f(a)$.

Continuity of Some Common Functions

Function $f(x)$	Interval in which $f(x)$ is continuous
Constant function : $f(x) = C$	R
Identity function : $f(x) = x$	R
Polynomial function : $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$	R
Rational function : $f(x) = \frac{p(x)}{q(x)}; q(x) \neq 0$	$R - \{x : q(x) = 0\}$
Modulus function : $f(x) = x $	R
x^{-n}, n is positive integer	$R - \{0\}$
$\sin x, \cos x, e^x$	R
$\tan x, \sec x$	$R - \{(2n+1)\frac{\pi}{2} : n \in Z\}$
$\cot x, \operatorname{cosec} x$	$R - \{n\pi : n \in Z\}$
$\log x$	$(0, \infty)$

9. The function $f(x) = \sqrt{3} \sin 2x - \cos 2x + 4$ is one-one in the interval

- A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ B. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
C. $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$ D. $[-\pi, \pi]$

10. If the points $(3, -2), (x, 2), (8, 8)$ are collinear, then find the value of x .

- A. 2 B. 3 C. 4 D. 5

11. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, $0 < x \leq 1$, then in the interval,

- A. both $f(x)$ and $g(x)$ are increasing functions
B. both $f(x)$ and $g(x)$ are decreasing functions
C. $f(x)$ is an increasing function
D. $g(x)$ is an increasing function

12. The value of $\int \cos 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$ is equal to

- A. $\ln \sqrt{\sin 2\theta} + \sin 2\theta \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} + c$
B. $\ln \sqrt{\cos 2\theta} + \sin 2\theta \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} + c$
C. $-\ln \sqrt{\cos 2\theta} - \sin 2\theta \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} + c$
D. $\ln \sqrt{\sin 2\theta} - \cos 2\theta \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} + c$

13. Maximize $Z = 4x + 6y$, subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$.

- A. 16 at $(4, 0)$ B. 24 at $(0, 4)$
C. 24 at $(6, 0)$ D. 36 at $(0, 6)$

14. One-thirds of the students of a class are boys and the rest are girls. It is known that the probability of a girl getting a first class marks in Board's Exam is 0.6 and a boy getting first class marks is 0.35. Find the probability that a student chosen at random will get first class marks in exam.

- A. $\frac{37}{60}$ B. $\frac{33}{50}$ C. $\frac{31}{60}$ D. $\frac{31}{50}$

15. The solution of differential equation

$$\frac{dy}{dx} = e^{x-y} (e^x - e^y) \text{ is}$$

- A. $e^y = e^x + 1 + ce^{-e^x}$ B. $e^y = e^x + ce^{e^x}$
C. $e^{y-x} = ce^{-e^x} - 1$ D. $e^y = e^x - 1 + ce^{-e^x}$

ACHIEVERS SECTION

16. Read the given statements carefully and state 'T' for true and 'F' for false.

- (i) If one of the roots of the equation

$$\begin{vmatrix} 7 & 6 & x^2 - 13 \\ 2 & x^2 - 13 & 2 \\ x^2 - 13 & 3 & 7 \end{vmatrix} = 0 \text{ is } -2, \text{ then sum}$$

of all other five roots is 5.

(ii) The value of $\begin{vmatrix} \sqrt{6} & 2i & 3 + \sqrt{6} \\ \sqrt{12} & \sqrt{3} + \sqrt{8}i & 3\sqrt{2} + \sqrt{6}i \\ \sqrt{18} & \sqrt{2} + \sqrt{12}i & \sqrt{27} + 2i \end{vmatrix}$

is -6.

(iii) If $f(\theta) = \begin{vmatrix} \cos^2 \theta & \cos \theta \sin \theta & -\sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & \cos \theta \\ \sin \theta & -\cos \theta & 0 \end{vmatrix}$, then

the value of $f(\pi/3)$ is 1.

mtg

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	(i)	(ii)	(iii)
A.	T	F	F
B.	F	T	T
C.	T	T	F
D.	F	F	T

17. Fill in the blanks and select the correct option.

(i) If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2 \tan^{-1} x$,
then x is equal to (P).

(ii) $\cos\left\{\tan^{-1}\left(\tan\frac{15\pi}{4}\right)\right\} = \text{---(Q)---}$.

(iii) The number of real solutions of the equation
 $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$, is
(R).

	(P)	(Q)	(R)
A.	$\frac{a-b}{1+ab}$	$\frac{1}{\sqrt{2}}$	1
B.	$\frac{b}{1-ab}$	$\sqrt{2}$	0
C.	$\frac{a+b}{1-ab}$	$\frac{1}{\sqrt{2}}$	2
D.	$\frac{a-b}{1+ab}$	2	2

18. Read the following statements carefully and select the correct option.

Statement-1 : If $\operatorname{cosec}^{-1}x + \operatorname{cosec}^{-1}y + \operatorname{cosec}^{-1}z = \frac{-3\pi}{21}$, then value of $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ is 3.

Statement-2 :

$\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\alpha\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\alpha\right\}$,
($\alpha \neq 0$) is equal to $\frac{2}{\alpha}$.

- A. Both Statement-1 and Statement-2 are true.
B. Both Statement-1 and Statement-2 are false.
C. Statement-1 is true but Statement-2 is false.
D. Statement-1 is false but Statement-2 is true.

19. Match the following and select the correct option.

Column I		Column II	
(i)	The domain of the function $f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi+x)\}}}$ where $\{\cdot\}$ denotes fractional part, is	(p)	$[0, 1)$
(ii)	The range of the function $f(x) = \frac{1}{2 - \sin 3x}$, is	(q)	$R - \left[\frac{n\pi}{2}, n \in I\right]$
(iii)	If the function $f : R \rightarrow A$ defined by $f(x) = \frac{x^2}{x^2+1}$ is onto, then $A =$	(r)	$\left[\frac{1}{3}, 1\right]$

- A. (i) $\rightarrow$ (q); (ii) $\rightarrow$ (r); (iii) $\rightarrow$ (p)
B. (i) $\rightarrow$ (p); (ii) $\rightarrow$ (r); (iii) $\rightarrow$ (q)
C. (i) $\rightarrow$ (q); (ii) $\rightarrow$ (p); (iii) $\rightarrow$ (r)
D. (i) $\rightarrow$ (p); (ii) $\rightarrow$ (q); (iii) $\rightarrow$ (r)

20. A curve is such that the area of the region bounded by the co-ordinate axes, the curve & the ordinate of any point on it is equal to the cube of that ordinate.

The curve represents

- A. a pair of straight lines
B. a circle
C. a parabola
D. an ellipse

Darken your choice with HB Pencil

1. (A) (B) (C) (D)	5. (A) (B) (C) (D)	9. (A) (B) (C) (D)	13. (A) (B) (C) (D)	17. (A) (B) (C) (D)
2. (A) (B) (C) (D)	6. (A) (B) (C) (D)	10. (A) (B) (C) (D)	14. (A) (B) (C) (D)	18. (A) (B) (C) (D)
3. (A) (B) (C) (D)	7. (A) (B) (C) (D)	11. (A) (B) (C) (D)	15. (A) (B) (C) (D)	19. (A) (B) (C) (D)
4. (A) (B) (C) (D)	8. (A) (B) (C) (D)	12. (A) (B) (C) (D)	16. (A) (B) (C) (D)	20. (A) (B) (C) (D)

SOLUTIONS

1. (A): $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2} \cdot \frac{\left(\frac{y}{x}\right)^2 - 2 \cdot \left(\frac{y}{x}\right) - 1}{\left(\frac{y}{x}\right)^2 + 2 \cdot \left(\frac{y}{x}\right) - 1} \dots(i)$

Let, $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

From (i), we get $v + x \frac{dv}{dx} = \frac{v^2 - 2v - 1}{v^2 + 2v - 1}$
 $\Rightarrow x \frac{dv}{dx} = \frac{v^2 - 2v - 1}{v^2 + 2v - 1} - v \Rightarrow x \frac{dv}{dx} = \frac{-(v^3 + v^2 + v + 1)}{v^2 + 2v - 1}$

$\Rightarrow \int \left(\frac{-1}{v+1} + \frac{2v}{v^2+1} \right) dv = -\int \frac{dx}{x}$

Integrating both sides, we get

$-\log|v+1| + \log|1+v^2| = -\log|x| + \log c$

$\Rightarrow \log \left[\frac{v^2+1}{v+1} \right] = \log \left(\frac{c}{x} \right) \Rightarrow \frac{v^2+1}{v+1} = \frac{c}{x}$

$\Rightarrow \frac{y^2+x^2}{y+x} = c \Rightarrow (x^2+y^2)k = x+y$, where $k = \frac{1}{c}$

At $x = -1, y = -1 \Rightarrow k = 0$

So, $x + y = 0$, which is the equation of straight line.

2. (A)

3. (A): Expanding along R_1 , we get

$\Delta = \cos^2(\alpha + \beta) + \sin^2(\alpha + \beta) + \cos 2\beta = 1 + \cos 2\beta$.

Hence, independent of α .

4. (C): Since, three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent.

$\therefore \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

$\Rightarrow p(rq - p^2) - q(q^2 - pr) + r(qp - r^2) = 0$

$\Rightarrow pqr - p^3 - q^3 + pqr + pqr - r^3 = 0$

$\Rightarrow 3pqr = p^3 + q^3 + r^3$

5. (D): $x(1-x^2)dy + (2x^2y - y - ax^3)dx = 0$

$\Rightarrow x(1-x^2)\frac{dy}{dx} + 2x^2y - y - ax^3 = 0$

$\Rightarrow x(1-x^2)\frac{dy}{dx} + 2x^2y - y = ax^3$

$\Rightarrow \frac{dy}{dx} + \frac{y(2x^2-1)}{x(1-x^2)} = \frac{ax^3}{x(1-x^2)}$, which is of the form

$\frac{dy}{dx} + P = Q$, where $P = \frac{2x^2-1}{x(1-x^2)}$

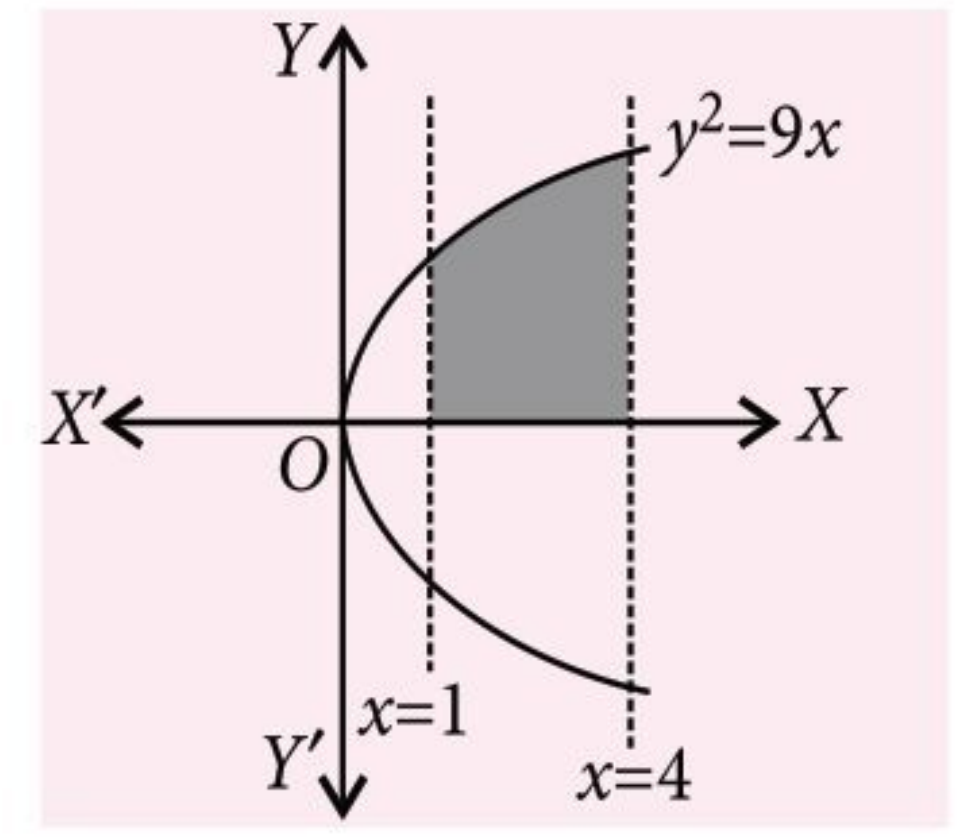
6. (B): We have, $y^2 = 9x$, a parabola

Required area

$= \int_1^4 3\sqrt{x} dx = 3 \left[\frac{x^{3/2}}{3/2} \right]_1^4$

$= 2(4^{3/2} - 1) = 2(8 - 1)$

$= 14$ sq. units



7. (A)

8. (A): $|A| = 4, |B| = 1024$

As, $B = \text{adj}(2A) \Rightarrow |\text{adj}(2A)| = 1024$

$\Rightarrow |2A|^{n-1} = 1024 \quad \left[\because |\text{adj } A| = |A|^{n-1} \right]$

$\Rightarrow (2^n \cdot 4)^{n-1} = 1024 \quad \left[\because |KA| = K^n |A| \right]$

$\Rightarrow 2^{(n+2)(n-1)} = 2^{10} \Rightarrow (n+2)(n-1) = 10$

$\Rightarrow n^2 + n - 12 = 0 \Rightarrow n = \frac{-1 \pm \sqrt{1+48}}{2} \Rightarrow n = 3, -4$

Hence, $n = 3$

$[\because x = -4, \text{ not possible}]$

9. (C): $f(x) = 2 \sin \left(2x - \frac{\pi}{6} \right) + 4$

$\therefore \sin x$ is one-one in $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$.

$\therefore -\frac{\pi}{2} \leq 2x - \frac{\pi}{6} \leq \frac{\pi}{2} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3} \right]$

10. (D): Since, the given points are collinear.

$\therefore \frac{1}{2} \begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$

$\Rightarrow 3(2-8) + 2(x-8) + 1(8x-16) = 0$

$\Rightarrow -18 + 2x - 16 + 8x - 16 = 0 \Rightarrow 10x = 50 \Rightarrow x = 5$

11. (C): $f'(x) = \frac{\sin x - x \cos x}{\sin^2 x}, g'(x) = \frac{\tan x - x \sec^2 x}{\tan^2 x}$

Now, $\frac{d}{dx}(\sin x - x \cos x) = x \sin x > 0$ for $0 < x \leq 1$

$\therefore \sin x - x \cos x$ is an increasing function.

Also, at $x = 0$, $\sin x - x \cos x = 0$

$\Rightarrow 0 < x \leq 1, \sin x - x \cos x > 0$

$\therefore f'(x) > 0$ for $0 < x \leq 1$

$\therefore f(x)$ is increasing on the interval $(0, 1]$.

Again, $\frac{d}{dx}(\tan x - x \sec^2 x) = -x \cdot 2 \sec^2 x \cdot \tan x < 0$

for $0 < x \leq 1$

$\Rightarrow \tan x - x \sec^2 x$ is a decreasing function.

Also at $x = 0$, $\tan x - x \sec^2 x = 0$

$\Rightarrow 0 < x \leq 1, \tan x - x \sec^2 x < 0$

$g'(x) < 0$ for $0 < x \leq 1$.

$\therefore g(x)$ is decreasing in $(0, 1]$.

12. (C) : We have, $\int \cos 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$

$$\begin{aligned} \text{Let } u &= \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \\ v &= \cos 2\theta \\ &= \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \int \cos 2\theta d\theta - \int \frac{2}{\cos 2\theta} \left(\frac{-\sin 2\theta}{2} \right) d\theta \\ &= \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \left(\frac{-\sin 2\theta}{2} \right) + \int \frac{\sin 2\theta}{\cos 2\theta} d\theta \\ &= -\frac{1}{2} \ln \left[\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right] \cdot \sin 2\theta - \ln \sqrt{\cos 2\theta} + c \\ &= -\sin 2\theta \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} - \ln \sqrt{\cos 2\theta} + c \\ &= -\ln \sqrt{\cos 2\theta} - \sin 2\theta \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} + c \end{aligned}$$

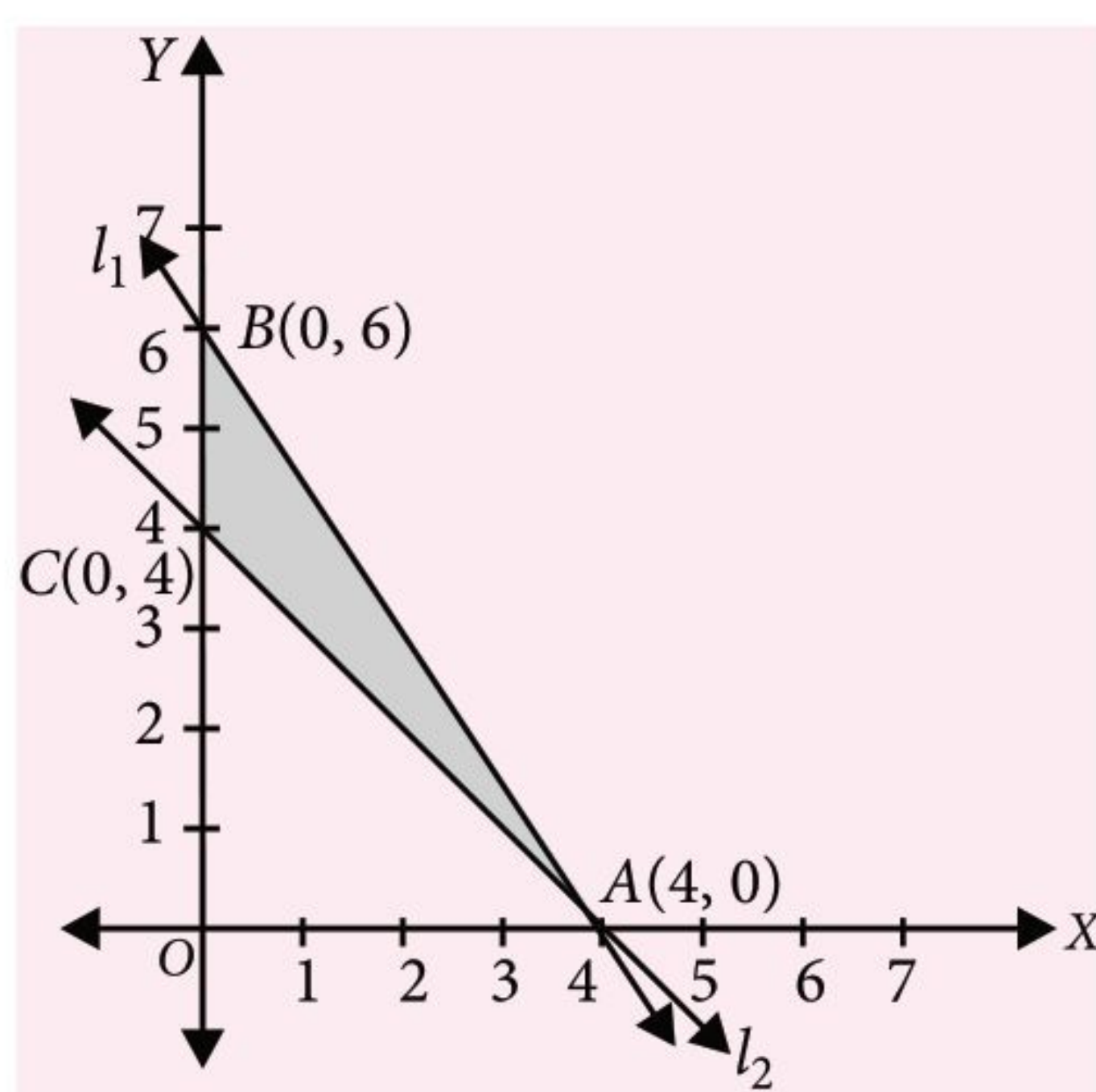
13. (D) : We have, maximize $Z = 4x + 6y$

Subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$

Let $l_1 : 3x + 2y = 12$

$l_2 : x + y = 4$; $l_3 : x = 0$ and $l_4 : y = 0$

Shaded portion ABC is the feasible region, where $A(4, 0)$, $B(0, 6)$, $C(0, 4)$.



Now maximize $Z = 4x + 6y$

Z at $A(4, 0) = 4(4) + 6(0) = 16$

Z at $B(0, 6) = 4(0) + 6(6) = 36$

Z at $C(0, 4) = 4(0) + 6(4) = 24$

Thus, Z is maximized at $B(0, 6)$ and its maximum value is 36.

14. (C) : Let E_1 , E_2 and A be the events defined as follows:

E_1 : a boy is chosen, E_2 : a girl is chosen and

A : the student gets first class marks

Then, $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{2}{3}$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

$P(A|E_1)$ = probability of a boy getting first class marks

$$= 0.35 = \frac{35}{100} = \frac{7}{20}$$

$P(A|E_2)$ = probability of a girl getting first class marks

$$= 0.6 = \frac{6}{10} = \frac{3}{5}$$

By using law of total probability, we get

$$P(A) = P(E_1) P(A|E_1) + P(E_2) P(A|E_2)$$

$$= \frac{1}{3} \cdot \frac{7}{20} + \frac{2}{3} \cdot \frac{3}{5} = \frac{7}{60} + \frac{2}{5} = \frac{31}{60}$$

15. (D) : We have, $\frac{dy}{dx} = e^{x-y} (e^x - e^y)$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{2x}}{e^y} - e^x \Rightarrow e^y \frac{dy}{dx} + e^x e^y = e^{2x} \quad \dots(i)$$

$$\text{Put } e^y = v \Rightarrow e^y \frac{dy}{dx} = \frac{dv}{dx}$$

$$\text{Thus, (i) reduces to } \frac{dv}{dx} + e^x \cdot v = e^{2x}$$

$$\text{I.F.} = e^{\int e^x dx} = e^{e^x}$$

$$\text{Thus, solution is } v \cdot e^{e^x} = \int e^{e^x} \cdot e^x \cdot e^x dx + c$$

$$\text{Putting } e^x = t \Rightarrow e^x dx = dt$$

$$\therefore e^y \cdot e^{e^x} = e^{e^x} (e^x - 1) + c \Rightarrow e^y = e^x - 1 + ce^{-e^x}$$

$$\text{16. (D) : (i) } \begin{vmatrix} 7 & 6 & x^2 - 13 \\ 2 & x^2 - 13 & 2 \\ x^2 - 13 & 3 & 7 \end{vmatrix} = 0$$

Solving we, get $x^6 - 39x^4 + 440x^2 + 1200 = 0$

We know that, $ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + gx + f = 0$

Sum of all roots = $-b/a$

So, sum of all roots of $x^6 - 39x^4 + 440x^2 + 1200 = 0$,

$$\text{is } \left(-\frac{0}{1} \right) = 0$$

$$\Rightarrow -2 + \text{sum of other 5 roots} = 0$$

$$\Rightarrow \text{Sum of other 5 roots} = 2$$

$$\text{(ii) } \begin{vmatrix} \sqrt{6} & 2i & 3 + \sqrt{6} \\ \sqrt{12} & \sqrt{3} + \sqrt{8}i & 3\sqrt{2} + \sqrt{6}i \\ \sqrt{18} & \sqrt{2} + \sqrt{12}i & \sqrt{27} + 2i \end{vmatrix} = -6\sqrt{6}$$

$$\begin{aligned} \text{(iii) } f(\theta) &= \begin{vmatrix} \cos^2 \theta & \cos \theta \cdot \sin \theta & -\sin \theta \\ \cos \theta \cdot \sin \theta & \sin^2 \theta & \cos \theta \\ \sin \theta & -\cos \theta & 0 \end{vmatrix} \\ &= \cos^4 \theta + \cos^2 \theta \cdot \sin^2 \theta + \cos^2 \theta \cdot \sin^2 \theta + \sin^4 \theta \end{aligned}$$

$$f\left(\frac{\pi}{3}\right) = \cos^4\left(\frac{\pi}{3}\right) + 2\cos^2\left(\frac{\pi}{3}\right) \cdot \sin^2\left(\frac{\pi}{3}\right) + \sin^4\left(\frac{\pi}{3}\right) = 1$$

17. (C) : (i) $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1}x$

Put $a = \tan\alpha$, $b = \tan\beta$

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right) + \sin^{-1}\left(\frac{2\tan\beta}{1+\tan^2\beta}\right) = 2\tan^{-1}x$$

$$\Rightarrow \sin^{-1}(\sin 2\alpha) + \sin^{-1}(\sin 2\beta) = 2\tan^{-1}x$$

$$\Rightarrow 2\alpha + 2\beta = 2\tan^{-1}x$$

$$\Rightarrow 2[\tan^{-1}a + \tan^{-1}b] = 2\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1}x \Rightarrow x = \frac{a+b}{1-ab}$$

(ii) $\cos\left\{\tan^{-1}\left\{\tan\frac{15\pi}{4}\right\}\right\} = \cos\left\{\tan^{-1}\left\{\tan\left(4\pi - \frac{\pi}{4}\right)\right\}\right\}$
 $= \cos\left\{\tan^{-1}\left\{\tan\left(-\frac{\pi}{4}\right)\right\}\right\} = \frac{1}{\sqrt{2}}$

(iii) $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{L}$

Let, $y = \tan^{-1}\sqrt{x(x+1)}$

$$\tan y = \sqrt{x(x+1)}$$

We know that,

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\sqrt{x^2+x+1}\right) + \cos^{-1}\left(\frac{1}{\sqrt{x^2+x+1}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \sqrt{x^2+x+1} = \frac{1}{\sqrt{x^2+x+1}} \Rightarrow x^2+x+1 = 1$$

$$\Rightarrow x^2+x=0 \Rightarrow x(x+1)=0 \Rightarrow x=0, -1$$

The number of real solution is 2.

18. (A) : (i) We know that the minimum value of $\operatorname{cosec}^{-1}x$ is $-\frac{\pi}{2}$ which is attained at $x = -1$.

$$\therefore \operatorname{cosec}^{-1}x + \operatorname{cosec}^{-1}y + \operatorname{cosec}^{-1}z = -\frac{3\pi}{2}$$

$$\Rightarrow \operatorname{cosec}^{-1}x + \operatorname{cosec}^{-1}y + \operatorname{cosec}^{-1}z$$

$$= \left(-\frac{\pi}{2}\right) + \left(-\frac{\pi}{2}\right) + \left(-\frac{\pi}{2}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}x = -\frac{\pi}{2}, \operatorname{cosec}^{-1}y = -\frac{\pi}{2}, \operatorname{cosec}^{-1}z = -\frac{\pi}{2}$$

$$\Rightarrow x = -1, y = -1, z = -1$$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} = \frac{(-1)}{(-1)} + \frac{(-1)}{(-1)} + \frac{(-1)}{(-1)} = 3$$

(ii) $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\alpha\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\alpha\right],$

Let $\frac{1}{2}\cos^{-1}\alpha = t$; $\tan\left\{\frac{\pi}{4} + t\right\} + \tan\left\{\frac{\pi}{4} - t\right\}$

$$= \frac{1+\tan t}{1-\tan t} + \frac{1-\tan t}{1+\tan t} = \frac{2(1+\tan^2 t)}{1-\tan^2 t}$$

$$= \frac{2}{\cos 2t} = \frac{2}{\cos(\cos^{-1}\alpha)} = \frac{2}{\alpha}$$

19. (A) : (i)

$$f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi+x)\}}} = \frac{1}{\sqrt{\{\sin x\} + \{(-\sin x)\}}}$$

Now, $\{\sin x\} + \{-\sin x\} = \begin{cases} 0, & \sin x \text{ is integer} \\ 1, & \sin x \text{ is not integer} \end{cases}$

For $f(x)$ to be defined, $\{\sin x\} + \{-\sin x\} \neq 0$

$$\Rightarrow \sin x \neq \text{integer} \Rightarrow \sin x \neq \pm 1, 0 \Rightarrow x \neq \frac{n\pi}{2}$$

Hence, domain is $R - \left\{\frac{n\pi}{2}, n \in I\right\}$.

(ii) Range of function is $\left[\frac{1}{3}, 1\right]$

(iii) $f: R \rightarrow A$

$f(x) = \frac{x^2}{x^2+1}$ is onto

-	+	-
0		1

Let $y = \frac{x^2}{x^2+1}$; $yx^2 + y = x^2 \Rightarrow y = x^2(1-y)$

$$\Rightarrow x^2 = \frac{y}{1-y} \geq 0, y \neq 1; A = [0, 1)$$

20. (C) : Given, $\int_0^x f(x) dx = y^3$

Differentiating w.r.t. x , we get

$$f(x) = 3y^2 \cdot \frac{dy}{dx} \Rightarrow y = 3y^2 \frac{dy}{dx}$$

$$\Rightarrow y = 0 \text{ (rejected)}$$

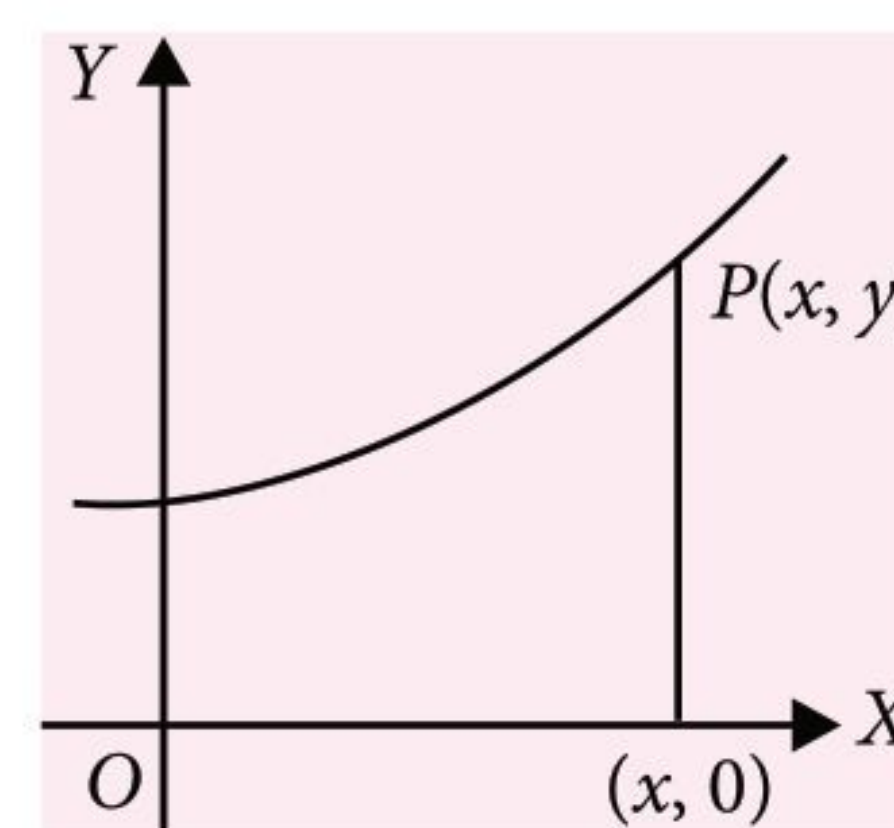
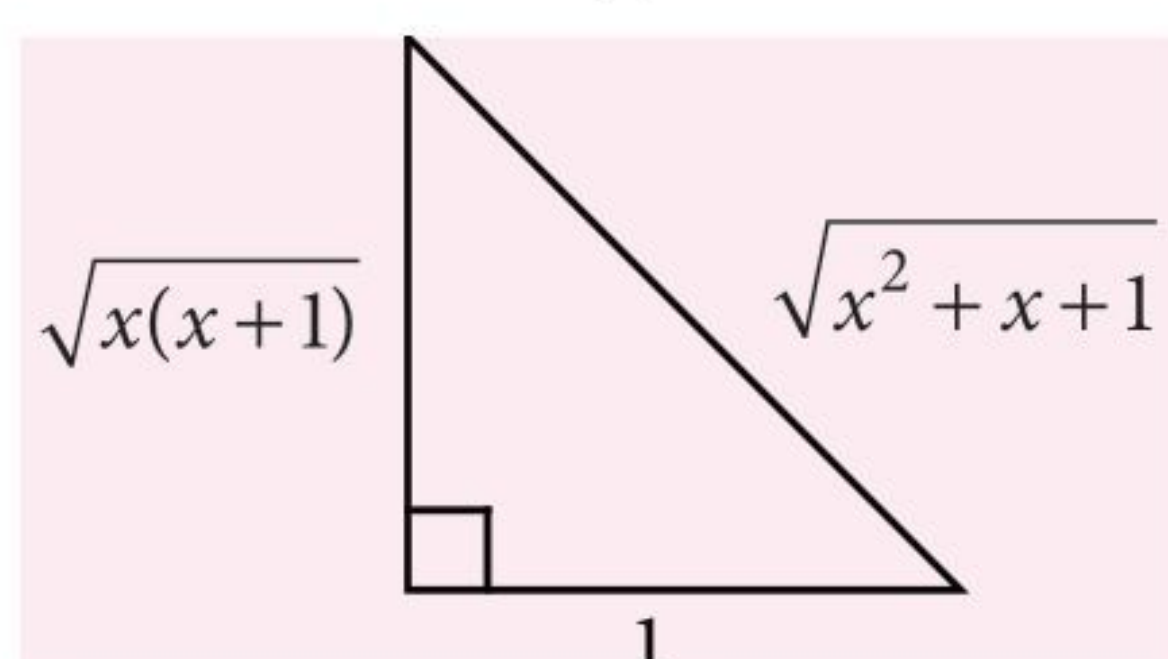
or $3ydy = dx$

Integrating both sides, we get,

$$\Rightarrow \frac{3y^2}{2} = x + c \Rightarrow y^2 = \frac{2}{3}x + k,$$

which represents a parabola.

For other sections/subjects please refer to
Physics For You, Chemistry Today and Biology Today



beat the TIME TRAP



Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. If $a = \cos\alpha + \cos\beta - \cos(\alpha + \beta)$,
 $b = 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\left(\frac{\alpha + \beta}{2}\right)$, then $(a - b)$ is equal to

- (a) 0 (b) 1 (c) -1 (d) -2

2. If α, β be the roots of $x^2 + x + 2 = 0$ and γ, δ be the roots of $x^2 + 3x + 4 = 0$, then $(\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$ is equal to

- (a) -18 (b) 18 (c) 24 (d) 44

3. A host invites 20 persons for a party. The number of ways they can be seated at a circular table, such that two particular persons be seated on either side of the host, is

- (a) 18! (b) $2 \times 18!$ (c) 19! (d) $2 \times 19!$

4. If $n \in \mathbb{N}$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by

- (a) 1904 (b) 2000 (c) 2002 (d) 2006

5. If the tangents to $y^2 = 4ax$ at the points (x_1, y_1) and (x_2, y_2) meet at the point (x_3, y_3) , then

- (a) $y_3 = y_1 + y_2$ (b) $y_3 = \sqrt{y_1 y_2}$
(c) $2y_3 = y_1 + y_2$ (d) $\frac{2}{y_3} = \frac{1}{y_1} + \frac{1}{y_2}$

6. If $A = \begin{pmatrix} 2 & 3-i & -i \\ 3+i & -5 & 7+i \\ i & 7-i & e \end{pmatrix}$, then A is

- (a) symmetric matrix
(b) hermitian matrix
(c) skew hermitian matrix
(d) none of these

7. Let $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then at $x = 0$, $f(x)$ is

- (a) unbounded
(b) bounded, but discontinuous
(c) continuous, but not differentiable
(d) differentiable.

8. Let $I = \int \frac{e^x dx}{e^{4x} + e^{2x} + 1}$, $J = \int \frac{e^{-x} dx}{e^{-4x} + e^{-2x} + 1}$.

Then for an arbitrary constant c , $J - I$ is equal to

- (a) $\frac{1}{2} \ln \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + c$ (b) $\frac{1}{2} \ln \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + c$
(c) $\frac{1}{2} \ln \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + c$
(d) $\frac{1}{2} \ln \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + c$

9. The solution of $\left(\frac{x+y-a}{x+y-b} \right) \frac{dy}{dx} = \frac{x+y+a}{x+y+b}$ is

- (a) $x - y + (b - a) \ln((x + y)^2 - ab) = c$
(b) $2(x - y) + (b - a) \ln((x + y)^2 - ab) = c$
(c) $y - x + (b - a) \ln((x + y)^2 - ab) = c$
(d) $2(y - x) + (b - a) \ln((x + y)^2 - ab) = c$

10. The shortest distance between the lines $2x + y + z - 1 = 0 = 3x + y + 2z - 2$, and $x = y = z$, is

- (a) $\frac{1}{\sqrt{2}}$ units (b) $\sqrt{2}$ units
(c) $\frac{3}{\sqrt{2}}$ units (d) $\frac{\sqrt{3}}{2}$ units

MONTHLY TEST DRIVE CLASS XI

ANSWER

KEY

1. (c) 2. (a) 3. (a) 4. (c) 5. (c)
6. (b) 7. (b,c) 8. (a,b,c,d) 9. (a,b) 10. (a)
11. (a,c,d) 12. (a,c) 13. (a,b) 14. (a) 15. (c)
16. (b) 17. (6) 18. (3) 19. (785) 20. (6)

SECTION-II

Numerical Answer Type

11. If $N = \sum_{r < s} (r - s)^2$, where $r, s \in \{1, 3, 5, \dots, 19\}$,

then the sum of the digits of N is _____.

12. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is _____.

13. If x, y, z are non-zero real numbers and

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0, \text{ then } -\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \text{_____}.$$

14. The chord of the curve $y = -a^2x^2 + 5ax - 4$ touches the curve $y = \frac{1}{1-x}$ at the point $x = 2$ and is bisected by that point. Then the value of a is _____.

15. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. If $\vec{a} = \lambda(\vec{b} \times \vec{c})$, then value of $|\lambda|$ is _____.

SOLUTIONS

$$\begin{aligned} 1. \quad (b) : b &= 4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \left(\frac{\alpha + \beta}{2} \right) \\ &= 2 \left(\cos \left(\frac{\alpha - \beta}{2} \right) - \cos \left(\frac{\alpha + \beta}{2} \right) \right) \cos \left(\frac{\alpha + \beta}{2} \right) \\ &= 2 \cos \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right) - 2 \cos^2 \left(\frac{\alpha + \beta}{2} \right) \\ &= \cos \alpha + \cos \beta - (1 + \cos (\alpha + \beta)) = a - 1 \\ \Rightarrow a - b &= 1 \end{aligned}$$

$$\begin{aligned} 2. \quad (d) : \alpha + \beta &= -1, \alpha\beta = 2, \gamma + \delta = -3, \gamma\delta = 4 \\ \therefore (\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta) &= (\alpha^2 - 3\alpha + 4)(\beta^2 - 3\beta + 4) \\ &= 4 - 3\alpha\beta^2 + 4\beta^2 - 3\alpha^2\beta + 9\alpha\beta - 12\beta + 4\alpha^2 - 12\alpha + 16 \\ &= 4 - 6\beta + 4(\alpha^2 + \beta^2) - 6\alpha + 18 + 12 + 16 \\ &= 50 + 6 + 4[(\alpha + \beta)^2 - 2\alpha\beta] \\ &= 56 + 4(1 - 4) = 56 - 12 = 44 \end{aligned}$$

3. (b) : The 18 guests excluding the two particular ones and the host sit in $18!$ ways. The two particular guests sit on either side of the host in 2 ways. The desired number of ways is $2 \times 18!$

4. (b) : $121^n - 25^n = (96 + 25)^n - 25^n$ is divisible by 96. $1900^n - (-4)^n = (1904 - 4)^n - (-4)^n$ is divisible by 1904. Hence both the above are divisible by 16.

Further, $121^n - (-4)^n$ is divisible by 125 and $1900^n - 25^n$ is divisible by 1875. Hence both the above are divisible by 125.

$\therefore$ The given number is divisible by $16 \times 125 = 2000$.

5. (c) : The tangent at $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ meet at $(at_1t_2, a(t_1 + t_2))$

$$\therefore y_3 = a(t_1 + t_2) = \frac{y_1 + y_2}{2} \text{ or } 2y_3 = y_1 + y_2.$$

6. (b)

7. (c) : $\frac{1}{|x|} + \frac{1}{x} = 0$ if $x < 0$ and $\frac{2}{x}$ for $x > 0$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x e^{-\frac{2}{x}} = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} x = 0 \text{ and } f(0) = 0$$

$\therefore f(x)$ is continuous at $x = 0$

$$f'(0^+) = \lim_{x \rightarrow 0^+} e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)} = \lim_{x \rightarrow 0^+} e^{-\frac{2}{x}} = 0$$

$$f'(0^-) = \lim_{x \rightarrow 0^-} e^0 = 1. f'(0) \text{ does not exist.}$$

$$8. \quad (c) : J - I = \int \frac{(e^{3x} - e^x) dx}{e^{4x} + e^{2x} + 1} = \int \frac{(t^2 - 1) dt}{t^4 + t^2 + 1} \quad (\text{putting } e^x = t)$$

$$= \int \frac{\left(1 - \frac{1}{t^2}\right) dt}{t^2 + 1 + \frac{1}{t^2}} = \int \frac{d\left(t + \frac{1}{t}\right)}{\left(t + \frac{1}{t}\right)^2 - 1}$$

$$= \frac{1}{2} \ln \left(\frac{t + \frac{1}{t} - 1}{t + \frac{1}{t} + 1} \right) + c = \frac{1}{2} \ln \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + c.$$

$$9. \quad (d) : x + y = z, \frac{dy}{dx} = \frac{dz}{dx} - 1$$

MONTHLY TEST DRIVE CLASS XII

ANSWER

KEY

1. (a) 2. (d) 3. (b) 4. (b) 5. (c)
6. (d) 7. (a,b,c) 8. (a,b) 9. (b,c) 10. (a,c,d)
11. (a,c) 12. (c) 13. (a,c,d) 14. (d) 15. (d)
16. (b) 17. (1) 18. (2) 19. (10) 20. (144)

$$\Rightarrow \left(\frac{z-a}{z-b} \right) \left(\frac{dz}{dx} - 1 \right) = \frac{z+a}{z+b}$$

$$\Rightarrow \left(\frac{z-a}{z-b} \right) \frac{dz}{dx} = \frac{z+a}{z+b} + \frac{z-a}{z-b} = \frac{2(z^2-ab)}{(z+b)(z-b)}$$

$$\Rightarrow \frac{(z-a)(z+b)}{z^2-ab} dz = 2dx$$

$$\Rightarrow \left(1 + \frac{(b-a)z}{z^2-ab} \right) dz = 2dx$$

$$\text{Integrating, } z + \frac{b-a}{2} \ln(z^2-ab) = 2x + \frac{c}{2}$$

$$\Rightarrow 2(y-x) + (b-a) \ln((x+y)^2-ab) = c.$$

10. (a) : Any plane passing through first line is $2x + y + z - 1 + \lambda(3x + y + 2z - 2) = 0$

Line $x = y = z$ is passing through the point $O(0, 0, 0)$.
Required shortest distance = distance of O from the member plane of above family which is parallel to the line $x = y = z$

If plane is parallel to the line,

$$(2 + 3\lambda)1 + (1 + \lambda)1 + (1 + 2\lambda)1 = 0 \Rightarrow \lambda = -\frac{2}{3}$$

Equation of plane is

$$3(2x + y + z - 1) - 2(3x + y + 2z - 2) = 0$$

$$\text{or } y - z + 1 = 0$$

Its distance from $(0, 0, 0)$ is $\frac{1}{\sqrt{2}}$.

11. (6)

12. (5) : The pair of lines is

$$3x^2 - xy + 3y^2 + (2x - 3y) \frac{(2x+3y)}{k} - 4 \frac{(2x+3y)^2}{k^2} = 0,$$

Now, coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow 3 + 3 + \frac{4-9}{k} - \frac{4(4+9)}{k^2} = 0$$

$$\Rightarrow 6k - \frac{52}{k} = 5$$

13. (3)

$$\mathbf{14. (1) : } y = \frac{1}{1-x}$$

At $x = 2, y = -1$

$$\frac{dy}{dx} = \frac{1}{(1-x)^2}, \quad \therefore \left[\frac{dy}{dx} \right]_{(2,-1)} = 1$$

So, equation of tangent to the curve is

$$y + 1 = 1(x - 2) \text{ or } y = x - 3$$

Solving this line with $y = -a^2x^2 + 5ax - 4$, we get

$$x - 3 = -a^2x^2 + 5ax - 4$$

$$\text{or } a^2x^2 + x(1 - 5a) + 1 = 0 \quad \dots(i)$$

Above equation has two roots x_1 and x_2 which are abscissas of point of intersection P and Q .

Mid point of PQ lies on $x = 2$

$$\therefore \frac{x_1 + x_2}{2} = 2 \Rightarrow x_1 + x_2 = 4$$

$$\text{From equation (i), } x_1 + x_2 = -\left(\frac{1-5a}{a^2} \right) \Rightarrow 4 = \frac{5a-1}{a^2}$$

$$\Rightarrow 4a^2 - 5a + 1 = 0 \Rightarrow a = 1/4, 1$$

But for $a = 1/4$, eq. (i) gives imaginary roots.

15. (2) : Given $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

$\vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{c}$.

$\therefore \vec{a}$ is perpendicular to the plane of $\vec{b}$ and $\vec{c}$.

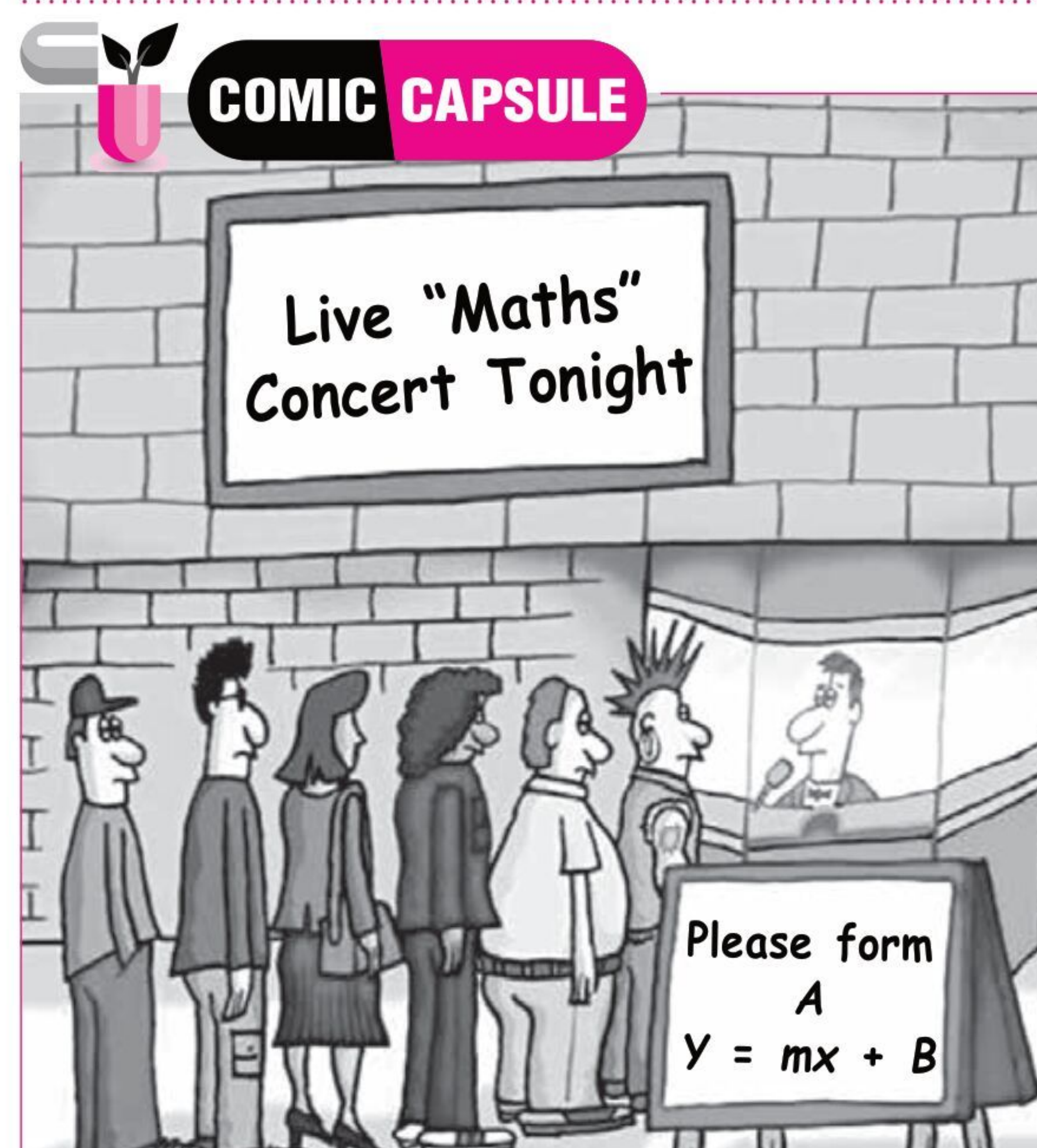
Also $\vec{a}$ is a unit vector.

$$\therefore \vec{a} = \pm \frac{\vec{b} \times \vec{c}}{|\vec{b} \times \vec{c}|} \quad \dots(i)$$

$$\text{But } |\vec{b} \times \vec{c}| = |\vec{b}| |\vec{c}| \sin \frac{\pi}{6} = 1 \cdot 1 \cdot \frac{1}{2}$$

$\therefore$ From (i), we have $\vec{a} = \pm 2(\vec{b} \times \vec{c})$

$$\therefore |\lambda| = 2.$$



Challenging PROBLEMS



ON

Coordinate Geometry



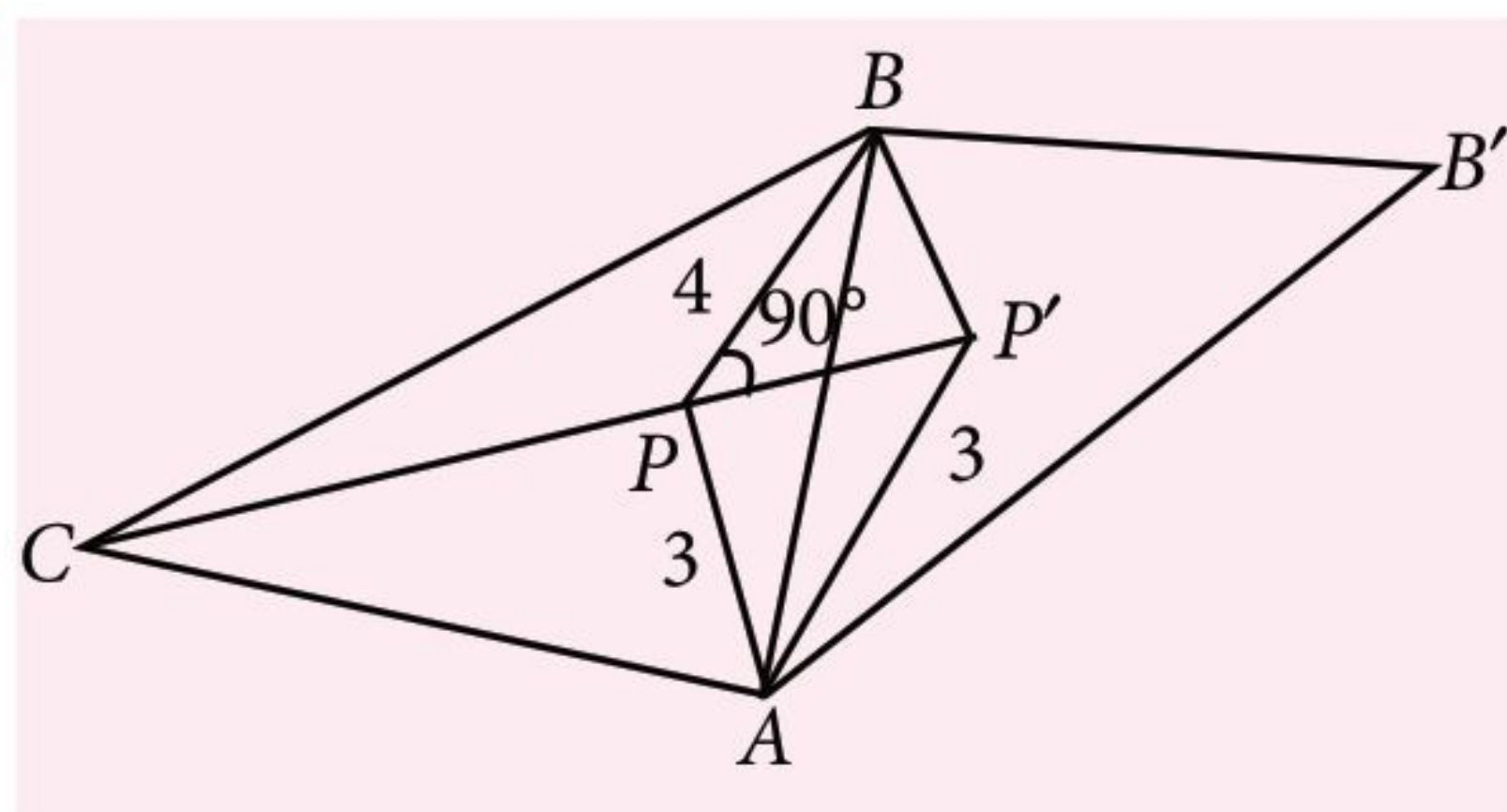
- If P be a point inside an equilateral $\triangle ABC$ such that $PA = 3$, $PB = 4$ and $PC = 5$, then the side length of the equilateral $\triangle ABC$ is
 (a) $\sqrt{25 - 12\sqrt{3}}$ (b) 13
 (c) $\sqrt{25 + 12\sqrt{3}}$ (d) 17
- In triangle ABC , equation of the side BC is $x - y = 0$. Circumcentre and orthocenter of the triangle are $(2, 3)$ and $(5, 8)$ respectively. Equation of the circumcircle of the triangle is
 (a) $x^2 + y^2 - 4x + 6y - 27 = 0$
 (b) $x^2 + y^2 - 4x - 6y - 27 = 0$
 (c) $x^2 + y^2 + 4x + 6y - 27 = 0$
 (d) $x^2 + y^2 + 4x - 6y - 27 = 0$
- Circles C_1 and C_2 having centres G_1 and G_2 respectively intersect each other at the points A and B , secants L_1 and L_2 are drawn to the circles C_1 and C_2 to intersect them in the points A_1, B_1 and A_2, B_2 respectively. If the secants L_1 and L_2 intersect each other at a point P in the exterior region of circles C_1 and C_2 and $PA_1 \times PB_1 = PA_2 \times PB_2$, then which of the following statement is false?
 (a) Points P, A and B are collinear
 (b) Line joining G_1 and G_2 is perpendicular to line joining P and A
 (c) $PA_1 \times PB_1 = PA \times PB$
 (d) $PA = PA_1$
- The locus of the centre of a circle which cuts orthogonally the parabola $y^2 = 4x$ at $(1, 2)$ will pass through points
 (a) $(3, 4)$ (b) $(4, 3)$ (c) $(5, 3)$ (d) $(2, 4)$
- Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90° at the point $(2, 3)$, then the locus of the midpoint of AB is circle whose centre is
 (a) $(1, 5)$ (b) $\left(1, \frac{5}{2}\right)$ (c) $\left(1, \frac{3}{2}\right)$ (d) $\left(2, \frac{5}{2}\right)$
- If a pair of variable straight lines $x^2 + 4y^2 + \alpha xy = 0$ (where α is a real parameter) cut the ellipse $x^2 + 4y^2 = 4$ at two points A and B , then locus of point of intersection of tangents at A and B is
 (a) $x^2 - 4y^2 + 8xy = 0$ (b) $(2x - y)(2x + y) = 0$
 (c) $x^2 - 4y^2 + 4xy = 0$ (d) $(x - 2y)(x + 2y) = 0$
- If directions of two sides of a triangle are fixed and length of third side is constant and is sliding between these sides, then locus of the orthocenter of the triangle is
 (a) circle (b) ellipse
 (c) straight line (d) hyperbola
- If an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has least area but contain the circle $(x - 1)^2 + y^2 = 1$, then
 (a) equation of ellipse is $2x^2 + 6y^2 = 9$
 (b) equation of ellipse is $6x^2 + 2y^2 = 9$
 (c) eccentricity of ellipse is $2e = \sqrt{\frac{2}{3}}$
 (d) eccentricity of ellipse is $e = 1/2$
- Consider two concentric circle $C_1 : x^2 + y^2 = 1$ and $C_2 : x^2 + y^2 - 4 = 0$. A parabola is drawn through the points where C_1 meet the x -axis and having arbitrary tangent of C_2 as its directrix. If locus of focus of drawn parabola is $\frac{3}{4}x^2 + y^2 = k$, then value of k is
 (a) 3 (b) 7 (c) 8 (d) 4
- All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ that subtend a right angle at the origin, pass through a fixed point (h, k) , then $h - k$ is equal to
 (a) 1 (b) 2 (c) 3 (d) 4

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11. How many tangents to the circle $x^2 + y^2 = 3$ are there which are normal to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$?
 (a) 3 (b) 2 (c) 1 (d) 0
12. If the ellipse $\frac{x^2}{a^2-3} + \frac{y^2}{a+4} = 1$ is inscribed in a square of side length $a\sqrt{2}$, then a is equal to
 (a) 4 (b) 2
 (c) 1 (d) none of these
13. The points of intersection of the two ellipse $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x^2 + 2y^2 - 20x - 12y + 35 = 0$
 (a) lie on a circle centered at $\left(\frac{8}{3}, 3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$.
 (b) lie on a circle centered at $\left(\frac{8}{3}, -3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{3}}$.
 (c) lie on a circle centered at $(8, 9)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$.
 (d) are not concyclic.
14. If the chord of contact of tangents from 3 points A , B and C to the circle $x^2 + y^2 = a^2$ are concurrent, then A , B and C will
 (a) be concyclic
 (b) be collinear
 (c) form the vertices of triangle
 (d) none of these
15. Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to CD and $AB = 2CD$. Let AD be perpendicular to AB and CD . If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is
 (a) 3 (b) 2 (c) $3/2$ (d) 1

SOLUTIONS

1. (c):



Rotate the triangle in clockwise direction through an angle 60° .

Let the points A , B , C and P will be A , B' , B and P' respectively after the rotation.

We have, $PA = P'A = 3$ and $\angle PAP' = 60^\circ \Rightarrow PP' = 3$.

Also $CP = BP' = 5$.

So $\triangle BPP'$ is right angle triangle with $\angle BPP' = 90^\circ$.

Now apply cosine rule in $\triangle BPA$, where

$\angle BPA = 90^\circ + 60^\circ = 150^\circ$, $PA = 3$ and $BP = 4$

we get $AB = \sqrt{25 + 12\sqrt{3}}$.

2. (b): Let coordinates of P be $(5, 8)$. Reflection of P in BC will lie on circumcircle.

$\therefore$ Equation of circumcircle is

$$(x-2)^2 + (y-3)^2 = (8-2)^2 + (5-3)^2$$

$$\Rightarrow x^2 + y^2 - 4x - 6y - 27 = 0$$

3. (d): Line joining PAB will be the radical axis of the two circles. So options, (a), (b) and (c) are correct statements.

4. (a): Tangent to parabola $y^2 = 4x$ at $(1, 2)$ will be the locus.

$$\text{i.e., } y \cdot 2 = 2(x+1) \Rightarrow y = x+1$$

5. (d): Let midpoint of AB is $M(h, k)$.

AB subtends angle 90° at $(2, 3)$.

$$\Rightarrow AM = MB$$

$$= \sqrt{(h-2)^2 + (k-3)^2}$$

$$\text{Also, } CM^2 + MB^2 = CB^2$$

$$\Rightarrow (h-2)^2 + (k-2)^2 + (h-2)^2 + (k-3)^2 = 4$$

$$\Rightarrow x^2 + y^2 - 4x - 5y + \frac{17}{2} = 0$$

6. (d): Let the point of intersection of tangents at A and B be $P(h, k)$, then equation of AB is

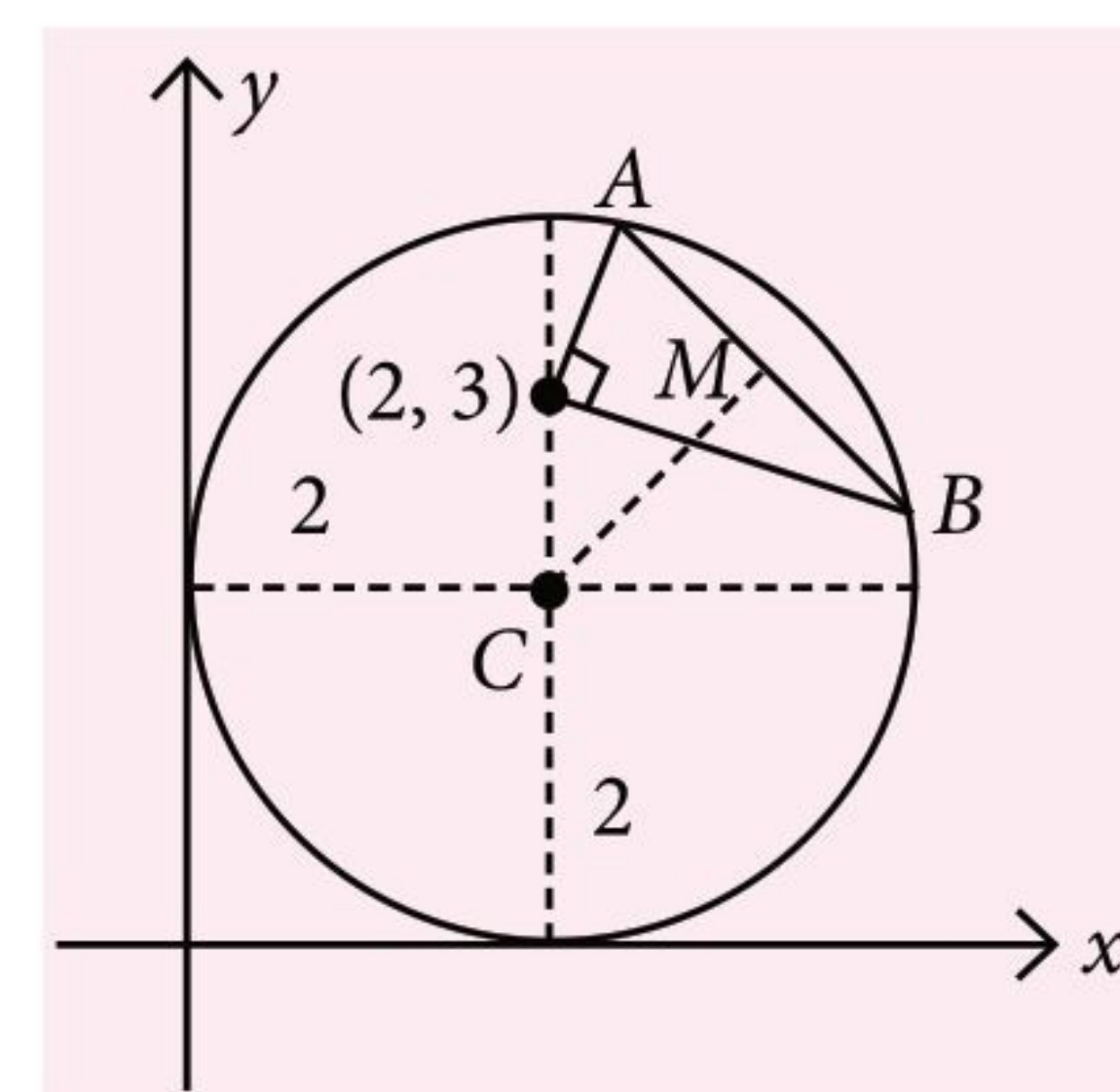
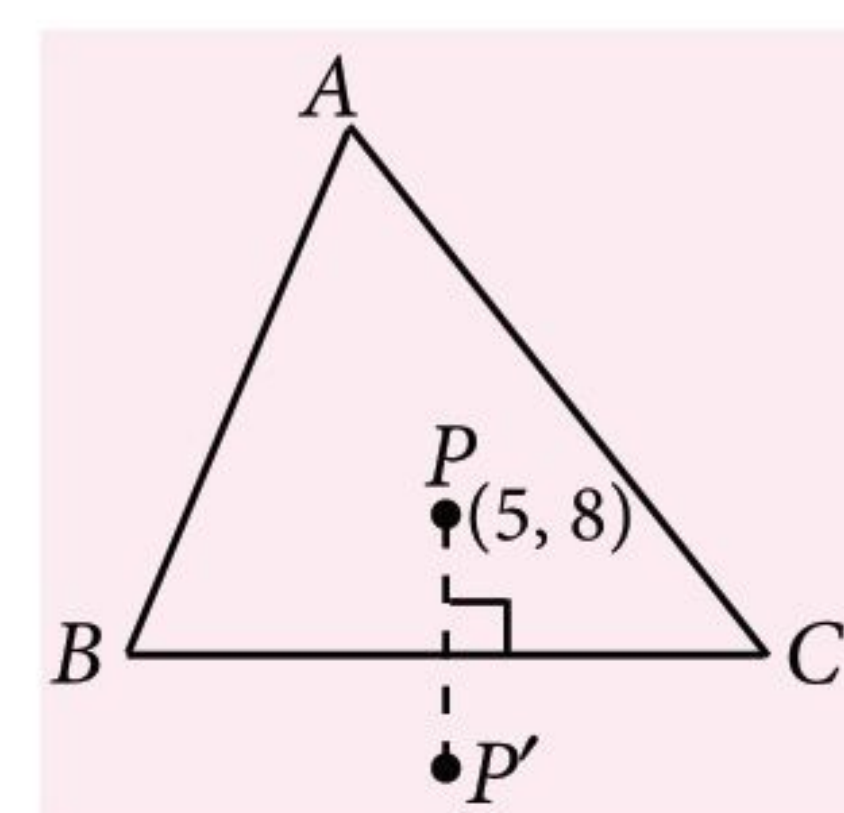
$$\frac{xh}{4} + \frac{yk}{1} = 1 \quad \dots (i)$$

Homogenizing the ellipse with (i), we get

$$\frac{x^2}{4} + \frac{y^2}{1} = \left(\frac{xh}{4} + \frac{yk}{1} \right)^2$$

$$\Rightarrow x^2 \left(\frac{h^2 - 4}{16} \right) + y^2 (k^2 - 1) + \frac{2hk}{4} xy = 0 \quad \dots (ii)$$

Given, equation of OA and OB is



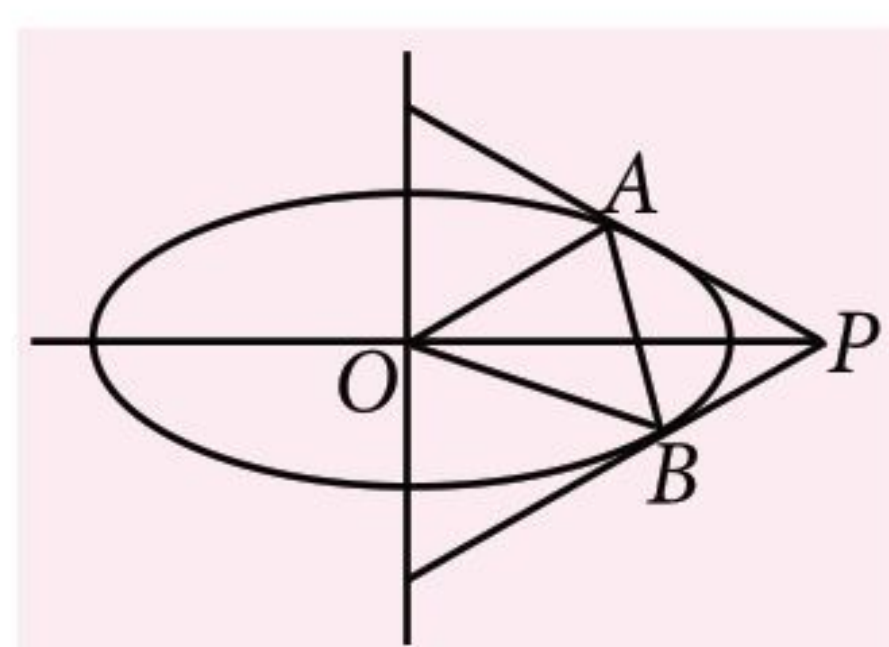
$$x^2 + 4y^2 + \alpha xy = 0 \quad \dots \text{(iii)}$$

Since, (ii) and (iii) are same

$$\therefore (h - 2k)(h + 2k) = 0$$

Thus, required locus is

$$(x - 2y)(x + 2y) = 0$$



7. (a): Let fixed directions be OA and OB inclined at a constant angle α and $AB = c$.

Let $\angle BAO = \theta$, then

$$BC = c \sin \theta \text{ and } AC = c \cos \theta.$$

$$\therefore OC = c \sin \theta \cdot \cot \alpha$$

Equation of the line passing through point A and perpendicular to OB is

$$y = -\cot \alpha (x - c \sin \theta \cot \alpha - c \cos \theta) \text{ and equation of BC is } x = c \sin \theta \cdot \cot \alpha$$

Orthocenter is $(c \sin \theta \cdot \cot \theta, c \cos \theta \cdot \cot \alpha)$

$\therefore$ Required locus is $x^2 + y^2 = c^2 \cot^2 \alpha$, which is the equation of a circle.

8. (a): On solving given conditions, we get

$$(b^2 - a^2)x^2 + 2a^2x - a^2b^2 = 0$$

$$\text{Now } D = 0 \Rightarrow a^2 - a^2e^2b^2 = 0 \Rightarrow b = 1/e$$

$$\text{Also, } a^2 = \frac{b^2}{1 - e^2} \Rightarrow a^2 = \frac{1}{e^2(1 - e^2)} \Rightarrow a = \frac{1}{e\sqrt{1 - e^2}}$$

$$\text{Now, } s = \pi ab = \frac{\pi}{e^2\sqrt{1 - e^2}}$$

$$\Rightarrow \frac{ds}{de} = \pi \left(\frac{e(3e^2 - 2)}{e^4(1 - e^2)^{3/2}} \right) \quad [\because 0 < e < 1]$$

$$\Rightarrow s \text{ is least, when } e = \sqrt{\frac{2}{3}}$$

$$\therefore \text{Equation of ellipse is } 2x^2 + 6y^2 = 9$$

$$\mathbf{9. (a):} (h - 1)^2 + k^2 = (\cos \theta - 2)^2 \quad \dots \text{(i)}$$

$$(h + 1)^2 + k^2 = (\cos \theta + 2)^2 \quad \dots \text{(ii)}$$

$$\text{(ii) - (i)} \Rightarrow \cos \theta = \frac{h}{2}$$

$$\text{(ii) + (i)} \Rightarrow 2(h^2 + k^2 + 1) = 2(\cos^2 \theta + 4) \Rightarrow \frac{3}{4}x^2 + y^2 = 3$$

10. (c): Let the equation of the chord is $y = mx + c$. Combined equation of the line joining the point of intersection with origin is

$$3x^2 - y^2 - 2(x - 2y) \left(\frac{y - mx}{c} \right) = 0$$

$$\Rightarrow x^2(3c + 2m) - y^2(c - 4) - 2xy(1 + 2m) = 0$$

From the condition of perpendicularity, we get

$$3c + 2m - c + 4 = 0 \Rightarrow m + c = -2$$

$\therefore$ The line $y = mx + c$, passes through $(1, -2)$.

11. (d): Equation of normal at $P(3\cos \theta, 2\sin \theta)$ is

$$3x \sec \theta - 2y \operatorname{cosec} \theta = 5$$

$$\frac{5}{\sqrt{9 \sec^2 \theta + 4 \operatorname{cosec}^2 \theta}} = \sqrt{3}$$

But minimum of $9 \sec^2 \theta + 4 \operatorname{cosec}^2 \theta = 25$

$\therefore$ No such θ exists.

12. (d): Sides of the square will be perpendicular tangents to the ellipse. So, vertices of the square will lie on the director circle. So, diameter of director circle is

$$2\sqrt{(a^2 - 3) + (a + 4)} = \sqrt{2a^2 + 2a^2}$$

$$\Rightarrow 2\sqrt{a^2 + a + 1} = 2a \Rightarrow a = -1$$

But for ellipse $a^2 > 3$ and $a > -4$

So, a cannot take the value -1 .

13. (a): If $S_1 = 0$ and $S_2 = 0$ are the equations, then $\lambda S_1 + S_2 = 0$ is a second degree curve passing through the points of intersection of $S_1 = 0$ and $S_2 = 0$.

$$\Rightarrow (\lambda + 4)x^2 + 2(\lambda + 1)y^2 - 2(3\lambda + 10)x - 12(\lambda + 1)y + (23\lambda + 35) = 0 \quad \dots \text{(i)}$$

For it to be a circle, choose λ such that the coefficients of x^2 and y^2 are equal.

$\therefore \lambda = 2$, which gives the equation of the circle as

$$6(x^2 + y^2) - 32x - 36y + 81 = 0 \quad \{\text{using (i)}\}$$

$$\Rightarrow x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0, \text{ whose}$$

Centre is $C\left(\frac{8}{3}, 3\right)$ and radius is

$$r = \sqrt{\frac{64}{9} + 9 - \frac{27}{2}} = \frac{1}{3}\sqrt{\frac{47}{2}}$$

14. (b): We have, $xx_1 + yy_1 = a^2$, $xx_2 + yy_2 = a^2$
an $xx_3 + yy_3 = a^2$

These lines will be concurrent, iff

$$\begin{vmatrix} x_1 & y_1 & -a^2 \\ x_2 & y_2 & -a^2 \\ x_3 & y_3 & -a^2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0,$$

which is the condition to the collinearity of points A, B and C.

15. (b): $A(0, 0)$; $B(2a, 0)$; $C(a, 2r)$; $D(0, 2r)$

$$\text{Area of quadrilateral } ABCD = \frac{1}{2}(2a + a) \times 2r = 18$$

$$\Rightarrow ar = 6 \quad \dots \text{(i)}$$

$$\text{Also, } a^2 + (2r)^2 = [(2a - r) + (a - r)]^2 \Rightarrow 8a^2 = 12ar$$

$$\Rightarrow a = (3/2)r \quad \dots \text{(ii)}$$

From (i) and (ii), we get $(3/2)r^2 = 6 \Rightarrow r = 2$





Based on CBSE Circular released on 16th Sept. 2022

CBSE warm-up!

IX-SSA17

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Series-3

Linear Inequalities | Permutations and Combinations | Binomial Theorem

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

- This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

- The marks obtained by a student of Class XI in first and second terminal examination are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have an average of at least 60 marks.
(a) 50 marks (b) 60 marks
(c) 70 marks (d) 80 marks
- The total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is
(a) 25 (b) 26 (c) 27 (d) 28
- If each multiple choice question has 4 choices, then the total number of ways of answering the 5 questions is
(a) 1100 (b) 1030 (c) 1020 (d) 1024
- The value of $\frac{n!}{r!(n-r)!}$, when $n = 5$, $r = 2$ is
(a) 30 (b) 10 (c) 20 (d) 15
- The value of x in $\frac{1}{x-2} < 0$ is
(a) $(-\infty, 2)$ (b) $(-\infty, \infty)$
(c) $(2, \infty)$ (d) $(-2, \infty)$
- The number of possible outcomes when a coin is tossed 6 times is
(a) 65 (b) 64 (c) 66 (d) 68
- Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is
(a) 10 (b) 14 (c) 12 (d) 8
- Solve the inequality : $2x - 5 \leq \frac{(4x - 7)}{3}$.
(a) $x \in (-\infty, 8)$ (b) $x \in (-\infty, 4]$
(c) $x \in (-\infty, 8]$ (d) $x \in (-\infty, 4)$
- If ${}^nP_r = 360$, then the sum of the values of n and r is
(a) 10 (b) 12 (c) 14 (d) 13

10. Three men have 4 coats, 5 waist coats and 6 caps. Then the number of ways they can wear them is
(a) 182000 (b) 162800
(c) 170000 (d) 172800
11. Using fundamental principle of counting, the number of 3-digit odd numbers is
(a) 400 (b) 450 (c) 500 (d) 550
12. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$, then the value of expression $a_2 + a_4 + a_6 + \dots + a_{12}$ is
(a) 40 (b) 41 (c) 30 (d) 31
13. Solve : $\frac{5-2x}{3} \leq \frac{x}{6} - 5$.
(a) $(8, \infty)$ (b) $[8, \infty)$ (c) $[-8, \infty]$ (d) $(-\infty, 8)$
14. If ${}^{10}C_{x-1} > 2 \cdot {}^{10}C_x$, then the set of values of x is
(a) 7, 8, 9 (b) 7, 9, 10
(c) 8, 9, 10 (d) 8, 10, 12
15. Ten buses are plying between two places A and B. Then the number of ways a person can travel from A to B and come back?
(a) 90 (b) 100 (c) 110 (d) 150
16. Find the solution of $(x-1)^2(x+4) < 0$.
(a) $x \in (-\infty, -4)$ (b) $x \in (-\infty, 4)$
(c) $x \in (4, \infty)$ (d) $x \in (-4, \infty)$
17. Find the solution of $|3-x| = 3-x$.
(a) $x < 6$ (b) $x > 6$ (c) $x \leq 3$ (d) $x \geq 3$
18. Find the value of ${}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11}$ is
(a) $2^{14} - 15$ (b) $2^{15} - 14$
(c) $2^{14} - 13$ (d) $2^{13} - 14$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A)** : Given 4 flags of different colours, then number of different signals can be generated, if a signal requires the use of 2 flags one below the other is 12.
Reason (R) : The sum of all 4 digit numbers that can be formed by using the digits 2, 4, 6, 8 (repetition of digits not allowed) is 133320.

20. **Assertion (A)** : The last four digits of the natural number 7^{100} is 0003.

Reason (R) : The value of $\left(x^2 - \sqrt{1-x^2}\right)^4 + \left(x^2 + \sqrt{1-x^2}\right)^4$ is $2x^8 - 12x^6 + 14x^4 - 4x^2 + 2$.

SECTION - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.
22. How many distinct 8 digit numbers can be formed from digits of the number 62270208?

OR

Check whether the given statement is true or not 'If some or all of n objects are taken at a time, then the number of combinations is $2^n - 1$.

23. Solve the inequation : $\frac{11-2x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}, x \in N$.
24. Find the value of n such that ${}^nP_5 = 42 \cdot {}^nP_3, n > 4$.

OR

Show that the last two digits of the number 3^{400} is 01.

25. Find the sum of the series
 $1 + \frac{1}{3^2} + \frac{1 \cdot 4}{1 \cdot 2} \cdot \frac{1}{3^4} + \frac{1 \cdot 4 \cdot 7}{1 \cdot 2 \cdot 3} \cdot \frac{1}{3^6} + \dots$

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.
27. Find the number of ways in which n distinct objects can be put into three different boxes so that no box remains empty.

OR

If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then find the values of n and r .

28. Solve the inequation : $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$

29. Without expanding, find the value of $(2x - 1)^5 + 5(2x - 1)^4(1 - x) + 10(2x - 1)^3(1 - x)^2 + 10(2x - 1)^2(1 - x)^3 + 5(2x - 1)(1 - x)^4 + (1 - x)^5$.

30. In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks and less than 80 marks, find the range of marks he should score in the fifth paper.

OR

Solve for x : $\frac{|x - 2| - 1}{|x - 2| - 2} \leq 0$

31. Show that $2^{4n+4} - 15n - 16$, where $n \in N$ is divisible by 225.

OR

If $(1 + ax)^n = 1 + 8x + 24x^2 + \dots$, then find the value of $a + n$.

SECTION - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Prove that $33!$ is divisible by 2^{15} . What is the largest integer n such that $33!$ is divisible by 2^n ?
33. Prove that $(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6 = 970$.

OR

Find the remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9.

34. Solve the following system of inequations:

$$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}, \frac{7x-1}{3} - \frac{7x+2}{6} > x$$

OR

Solve the inequation

$$|x - 1| + |x - 2| + |x - 3| \leq 6.$$

35. Find n , if $\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24 : 1$.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case study 1:** Read the following passage and answer the questions given below:

Ramesh travelled from India to America with his suitcase. After reaching to the hotel in America, he

tried to open his suitcase but forget the password of the lock. He remembered that there are 2 digits followed by 2 letters in the password.



- (i) How many possibilities are there for the password, if repetition is allowed?
- (ii) Find the possible number of passwords if neither words nor numbers are repeated.
- (iii) All of a sudden, Ramesh remembered that, the password starts with 4 and ends with H, then find the possible number of passwords if no entry is repeated.

OR

- (iii) How many different passwords are possible if he remembered first two and last entry?
37. **Case study 2:** Read the following passage and answer the questions given below:
In the evening, Riya started revision of the chapter "Binomial Theorem", which was taught by her class teacher in the school. But, she was unable to understand the relation of power and number of terms in a Binomial expansion. Her mother, who is also a Maths teacher helped her in clearing all her doubts by explaining the topic. Following are questions asked by Riya's mother.



- (i) Find the number of terms in $(2x - 7y)^7$.
- (ii) Find the number of terms in $((4x + y)^9 + (4x - y)^9)$.
- (iii) Find the number of terms in $((\sqrt{2} + 1)^{10} - (\sqrt{2} - 1)^{10})$.

OR

(iii) Find the number of terms in $(1 + x^2 + 2x)^{20}$.

38. Case study 3: Read the following passage and answer the questions given below:

A company produces certain items. The manager in the company maintain record about cost and revenue of these items on daily basis. The cost and revenue functions of product are given by $C(x) = 15x + 3000$ and $R(x) = 45x + 1500$ respectively, where x is the number of items produced and sold.

- (i) How many items must be sold to gain some profit?
- (ii) If number of items sold lies between 300 to 400 then what will be the revenue?

SOLUTIONS

1. (c) : Let x be the marks obtained by student in the annual examination. Then

$$\frac{62 + 48 + x}{3} \geq 60 \text{ or } 110 + x \geq 180 \text{ or } x \geq 70$$

Thus, the student must obtain a minimum of 70 marks to get an average of at least 60 marks.

2. (b) : Since the total number of terms are 52 of which 26 terms get cancelled. Therefore total number of terms in the given expansion after simplification is 26.

3. (d) : Since, each question can be answered in 4 ways. So, the total number of ways of answering 5 questions $= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 2^{10} = 1024$.

4. (b) : Since $n = 5, r = 2$

We have, $\frac{5!}{2!(5-2)!} = \frac{5!}{2! \times 3!} = \frac{4 \times 5}{2} = 10$

5. (a) : We have, $\frac{1}{x-2} < 0$
 $\Rightarrow x - 2 < 0 \quad \left[\because \frac{a}{b} < 0 \text{ and } a > 0 \Rightarrow b < 0 \right]$

$\Rightarrow x < 2 \Rightarrow x \in (-\infty, 2)$

Hence, the solution set of the given inequation is $(-\infty, 2)$.

6. (b) : We know, when a coin is tossed once then the number of possible outcomes is 2. (head(H) or tail(T)). So, in each successive toss the outcome will be H or T. Thus, each toss can give result in 2 ways. Hence, by multiplication principle required number of possible outcomes $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$.

7. (c) : Let the total number of persons be n .
 Since, total number of hand shakes = 66

$\therefore {}^nC_2 = 66 \Rightarrow \frac{n(n-1)}{2} = 66 \Rightarrow (n-12)(n+11) = 0$
 $\Rightarrow n = 12$ ($\because n$ cannot be negative)

8. (b) : We have, $2x - 5 \leq \frac{4x-7}{3}$
 $\Rightarrow 6x - 15 \leq 4x - 7 \Rightarrow 2x \leq 8$
 $\Rightarrow x \leq 4 \Rightarrow x \in (-\infty, 4]$

9. (a) : Given ${}^nP_r = 360$
 $\Rightarrow \frac{n!}{(n-r)!} = 360 = 6 \times 5 \times 4 \times 3$
 $= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{6!}{2!} = \frac{6!}{(6-4)!}$

On comparing, we get $n = 6$ and $r = 4$.

10. (d) : Required number of ways $= {}^4P_3 \times {}^5P_3 \times {}^6P_3$
 $= \frac{4!}{1!} \times \frac{5!}{2!} \times \frac{6!}{3!} = 4! \times (5 \times 4 \times 3) \times (6 \times 5 \times 4) = 172800$.

11. (b) : For the number to be odd, unit's place should be 1, 3, 5, 7 or 9.

So, number of ways of filling unit's place = 5

Now, number of ways of filling ten's place = 10 and number of ways of filling hundred's place = 9.

$\therefore$ Required number of 3-digit odd number
 $= 9 \times 10 \times 5 = 450$.

12. (d) : $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$
 On putting $x = 1$ and $x = -1$ in the given expansion and adding the results, we get

$64 = 2(1 + a_2 + a_4 + \dots + a_{12})$
 $\Rightarrow a_2 + a_4 + a_6 + \dots + a_{12} = 31$

13. (b) : We have, $\frac{5-2x}{3} \leq \frac{x}{6} - 5$

$\Rightarrow \frac{5-2x}{3} \leq \frac{x-30}{6}$
 $\Rightarrow 2(5-2x) \leq x-30 \Rightarrow 10-4x \leq x-30 \Rightarrow -5x \leq -40$
 $\Rightarrow x \geq 8$
 $\therefore$ Solution set is $\{x : x \geq 8, x \in R\}$ i.e., $[8, \infty)$.

14. (c) : $\frac{10!}{(x-1)!(11-x)!} > 2 \cdot \frac{10!}{(10-x)!x!}, 1 \leq x \leq 10$
 $\Rightarrow \frac{1}{11-x} > 2 \cdot \frac{1}{x} \Rightarrow x > 22 - 2x \Rightarrow x > \frac{22}{3}$.

Hence, $x = 8, 9, 10$.

15. (b) : There are 10 ways to travel from A to B. [A person can choose any bus out of 10]. Similarly, he can come back in 10 ways.

$\therefore$ Total number of ways to travel from A to B and come back = $10 \times 10 = 100$.

16. (a) : $(x - 1)^2$ is always positive except when $x = 1$ (and then it is 0)

$\therefore$ Solution is when $x + 4 < 0$ and $x \neq 1$

$\Rightarrow x < -4, x \neq 1 \therefore x \in (-\infty, -4).$

17. (c) : $|3 - x| = 3 - x$ is true only when $3 - x \geq 0$
i.e., iff $3 \geq x$ or $x \leq 3$.

18. (d) : ${}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11}$
 $= ({}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{13}) - {}^{14}C_{13}$
 $= 2^{14-1} - {}^{14}C_{13} = 2^{13} - 14$

19. (b) : Assertion (A) : There will be many signals as there are ways of filling 2 vacant places $\square\square$ in succession by the 4 flags of different colours. Hence, by the multiplication principle, the required number of signals $= 4 \times 3 = 12$.

Reason (R) : There are $4! = 24$ numbers. Each digit occurring $3! = 6$ times, in the unit's, ten's, hundred's and thousand's places. We note that $6(2 + 4 + 6 + 8) = 120$. Thus sum of digits at each place will be 120.

$\therefore$ The required sum $= 120 (1 + 10 + 10^2 + 10^3)$
 $= 120 \times 1111 = 133320$.

Both A and R are true but R is not the correct explanation of A.

20. (d) : Assertion (A) : $7^{100} = 49^{50} = (50 - 1)^{50} = (1 - 50)^{50}$
 $= 1 - 50 \times 50 + \frac{50 \cdot 49}{2} \cdot (50)^2 - \frac{50 \cdot 49 \cdot 48}{6} \cdot (50)^3$
 $\quad \quad \quad + \text{multiple of } 10^4$
 $= 1 - 2500 + 3062500 + \text{multiple of } 10^4$
 $= 3060001 + \text{multiple of } 10^4$
 $\therefore$ The last four digits are 0001.

Reason (R) : Putting $\sqrt{1 - x^2} = y$

The given expression becomes

$$\begin{aligned} &= (x^2 - y)^4 + (x^2 + y)^4 = 2[{}^4C_0 x^8 + {}^4C_2 x^4 y^2 + {}^4C_4 y^4] \\ &= 2[x^8 + 6x^4(1 - x^2) + (1 - 2x^2 + x^4)] \\ &= 2x^8 - 12x^6 + 14x^4 - 4x^2 + 2 \end{aligned}$$

A is false but R is true.

21. Let x be the smaller of the two consecutive odd natural numbers, so that the other one is $x + 2$. Then, we should have

$$x > 10 \text{ and } (x + 2) > 10 \quad \dots(i)$$

and $x + (x + 2) < 40$

$$\Rightarrow 2x + 2 < 40 \Rightarrow 2x < 38 \text{ i.e., } x < 19 \quad \dots(ii)$$

From (i) and (ii), we get $10 < x < 19$

Since x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be (11, 13), (13, 15), (15, 17) and (17, 19).

22. Number of ways of arranging the digits of the number 62270208 $= \frac{8!}{3!2!} = 3360$

But this also include those numbers which start with 0. If we fix 0 at first place then number of ways of arranging the digits of the number 62270208

$$= \frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840$$

$\therefore$ Required number of ways $= 3360 - 840 = 2520$.

OR

The given statement is true.

If some or all objects taken at a time, then number of selection is

$${}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$$

$$[\because {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n]$$

23. The given inequation is $\frac{11-2x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}$
 $\Rightarrow \frac{11-2x}{5} \geq \frac{9-3x+6}{8} \Rightarrow \frac{11-2x}{5} \geq \frac{15-3x}{8}$

$$\Rightarrow 8(11-2x) \geq 5(15-3x)$$

$$\Rightarrow 88 - 16x \geq 75 - 15x \Rightarrow 88 - 75 \geq -15x + 16x$$

$$\Rightarrow 13 \geq x \Rightarrow x \leq 13$$

Hence, the solution set is $\{1, 2, 3, \dots, 13\}$.

24. We have ${}^nP_5 = 42 {}^nP_3$
 $\Rightarrow n(n-1)(n-2)(n-3)(n-4) = 42 n(n-1)(n-2)$
 $\Rightarrow (n-3)(n-4) = 42$

[Since $n > 4$, so $n(n-1)(n-2) \neq 0$]

$$\Rightarrow n^2 - 7n - 30 = 0 \Rightarrow n^2 - 10n + 3n - 30 = 0$$

$$\Rightarrow (n-10)(n+3) = 0 \Rightarrow n-10 = 0 \text{ or } n+3 = 0$$

$$\Rightarrow n = 10 \text{ or } n = -3$$

As n cannot be negative, so $n = 10$.

OR

Since $3^{400} = 9^{200} = (10-1)^{200}$

$$= {}^{200}C_0 10^{200} - {}^{200}C_1 10^{199} + \dots - {}^{200}C_{199} 10^1 + {}^{200}C_{200} 1^{200}$$

$$= 10^{200} - (200 \times 10^{199}) + \dots - (10 \times 200) + 1$$

$$= 100k + 1, \text{ where } k \in N$$

Hence, last two digits are 01.

25. We know, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \dots + x^n$

$$\text{Here, } nx = \frac{1}{3^2} \quad \dots(i) \text{ and } \frac{n(n-1)}{2} x^2 = \frac{1 \cdot 4}{1 \cdot 2} \cdot \frac{1}{3^4} \quad \dots(ii)$$

From (i) and (ii), we get, $n = -\frac{1}{3}$ and $x = -\frac{1}{3}$

$$\therefore \text{ Required sum} = \left(1 - \frac{1}{3}\right)^{-1/3} = \left(\frac{2}{3}\right)^{1/3}$$

26. We have given, first pH value = 8.48
and second pH value = 8.35
Let third pH value be x .
Since, average pH values lies between 8.2 and 8.5.

$$\begin{aligned}\therefore 8.2 &< \frac{8.48 + 8.35 + x}{3} < 8.5 \\ \Rightarrow 8.2 &< \frac{16.83 + x}{3} < 8.5 \\ \Rightarrow 24.6 &< 16.83 + x < 25.5 \\ \Rightarrow 24.6 - 16.83 &< x < 25.5 - 16.83 \Rightarrow 7.77 < x < 8.67\end{aligned}$$

Hence, third pH value lies between 7.77 and 8.67.

27. Suppose first box remains empty, then for each of the n objects we have 2 choices. Thus number of ways in this case = $\underbrace{2 \times 2 \times \dots \times 2 \times 2}_{n \text{ times}} = 2^n$

Similarly if second or third box remains empty, this we get 2^n ways in each case.

Now, if first and second box remains empty, then there is only, one way of putting n distinct objects.

Similarly if second and third box or first and third box remains empty then we have 1 way in each case.

Hence, required number of cases
= $3^n - 2^n - 2^n - 2^n - 1 - 1 - 1 = 3^n - 3 \cdot 2^n - 3$.

OR

We have, ${}^nP_r = {}^nP_{r+1}$

$$\begin{aligned}\Rightarrow \frac{n!}{(n-r)!} &= \frac{n!}{(n-r-1)!} \\ \Rightarrow \frac{1}{(n-r)(n-r-1)!} &= \frac{1}{(n-r-1)!} \\ \Rightarrow n-r &= 1 \quad \dots(i)\end{aligned}$$

and, ${}^nC_r = {}^nC_{r-1}$

$$\begin{aligned}\Rightarrow \frac{n!}{(n-r)!r!} &= \frac{n!}{(n-r+1)!(r-1)!} \\ \Rightarrow \frac{n!}{(n-r)!r(r-1)!} &= \frac{n!}{(n-r+1)(n-r)!(r-1)!} \\ \Rightarrow \frac{1}{r} &= \frac{1}{n-r+1} \Rightarrow n-r+1 = r \\ \Rightarrow n-2r &= -1 \quad \dots(ii)\end{aligned}$$

On solving (i) and (ii), we get $n = 3$ and $r = 2$.

28. We have, $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3} \Rightarrow \frac{2x-3}{4} - \frac{4x}{3} \geq 3-9$

[Transposing $\frac{4x}{3}$ to L.H.S. and 9 to R.H.S.]

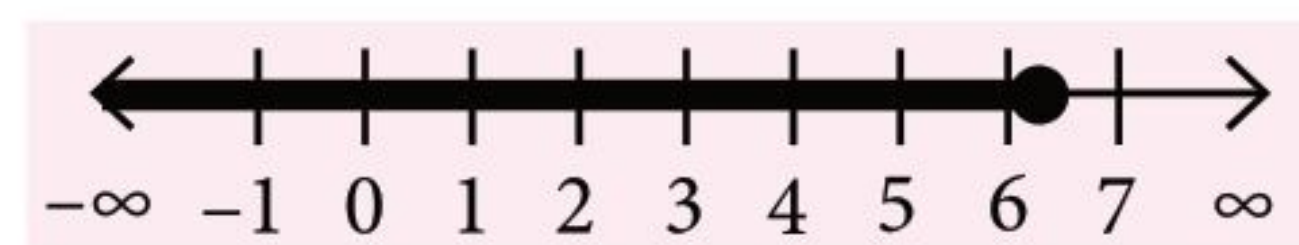
$$\begin{aligned}\Rightarrow \frac{3(2x-3)-16x}{12} &\geq -6 \\ \Rightarrow \frac{6x-9-16x}{12} &\geq -6 \Rightarrow \frac{-9-10x}{12} \geq -6\end{aligned}$$

$$\Rightarrow -9-10x \geq -72 \quad [\text{Multiplying both sides by 12}]$$

$$\Rightarrow -10x \geq -72+9 \Rightarrow -10x \geq -63$$

$$\Rightarrow \frac{-10x}{-10} \leq \frac{-63}{-10} \Rightarrow x \leq \frac{63}{10} \Rightarrow x \in (-\infty, 63/10]$$

Hence, the solution set of the given inequation is $(-\infty, 63/10]$. This can be graphed on real number line as shown below:



29. Clearly, $(2x-1)^5 + 5(2x-1)^4(1-x) + 10(2x-1)^3(1-x)^2 + 10(2x-1)^2(1-x)^3 + 5(2x-1)(1-x)^4 + (1-x)^5$
 $= {}^5C_0(2x-1)^5 + {}^5C_1(2x-1)^{5-1}(1-x) + {}^5C_2(2x-1)^{5-2}(1-x)^2 + {}^5C_3(2x-1)^{5-3}(1-x)^3 + {}^5C_4(2x-1)^{5-4}(1-x)^4 + {}^5C_5(2x-1)^{5-5}(1-x)^5$
 $= [(2x-1) + (1-x)]^5 = x^5$

30. Suppose Rishi scores x marks in the fifth paper, then $75 \leq \frac{95+72+73+83+x}{5} < 80$

$$\Rightarrow 75 \leq \frac{323+x}{5} < 80 \Rightarrow 375 \leq 323+x < 400$$

$$\Rightarrow 52 \leq x < 77$$

Hence, Rishi's score in fifth paper should be more than or equal to 52 and less than 77.

OR

$$\text{Let } |x-2| = y$$

$$\text{Then, } \frac{y-1}{y-2} \leq 0$$

Two cases arise :

$$(i) \quad y-1 \leq 0 \text{ and } y-2 > 0$$

$$(ii) \quad y-1 \geq 0 \text{ and } y-2 < 0$$

From (i), $y \leq 1$ and $y > 2$ (not possible)

From (ii), $y \geq 1$ and $y < 2 \quad \therefore 1 \leq y < 2$

$$\Rightarrow 1 \leq |x-2| < 2 \Rightarrow 1 \leq |x-2| \text{ and } |x-2| < 2$$

$$\Rightarrow \text{Either } x-2 \leq -1 \text{ or } x-2 \geq 1 \text{ and } -2 < x-2 < 2$$

$$\Rightarrow x \leq 1 \text{ or } x \geq 3 \quad \dots(i) \text{ and } 0 < x < 4 \quad \dots(ii)$$

$$\Rightarrow x \in (0, 1] \cup [3, 4). \quad [\text{From (i) and (ii)}]$$

31. We have, $2^{4n+4} - 15n - 16 = 2^{4(n+1)} - 15n - 16$
 $= 16^{n+1} - 15n - 16 = (1+15)^{n+1} - 15n - 16$
 $= {}^{n+1}C_0 + {}^{n+1}C_1 15^1 + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15^3 + \dots$
 $\quad + {}^{n+1}C_{n+1} (15)^{n+1} - 15n - 16$
 $= 1 + 15(n+1) + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15^3 + \dots$
 $\quad + {}^{n+1}C_{n+1} (15)^{n+1} - 15n - 16$
 $= 15^2 [{}^{n+1}C_2 + {}^{n+1}C_3 15 + \dots + {}^{n+1}C_{n+1} (15)^{n-1}]$

Thus, $2^{4n+4} - 15n - 16$ is divisible by 225.

OR

We know, $(1 + ax)^n = 1 + {}^nC_1(ax) + {}^nC_2(ax)^2 + \dots + {}^nC_n(ax)^n$

$$\therefore {}^nC_1(ax) = 8x \text{ and } {}^nC_2(ax)^2 = 24x^2$$

$$\Rightarrow nax = 8x \text{ and } \frac{n(n-1)}{2} a^2 x^2 = 24x^2$$

$$\Rightarrow na = 8 \dots (i) \text{ and } n(n-1)a^2 = 48 \dots (ii)$$

$$\Rightarrow n(n-1) \frac{64}{n^2} = 48 \quad [\text{Using (i) and (ii)}]$$

$$\Rightarrow \frac{(n-1)}{n} = \frac{3}{4} \Rightarrow 4n - 4 = 3n$$

$$\Rightarrow n = 4 \Rightarrow a = 2 \quad [\text{Using (i)}]$$

$$\Rightarrow a + n = 4 + 2 = 6.$$

$$32. \quad 33! = 1 \cdot 2 \cdot 3 \dots 31 \cdot 32 \cdot 33$$

$$= (2 \cdot 4 \cdot 6 \dots 32) (1 \cdot 3 \cdot 5 \dots 33) = (2 \cdot 1)(2 \cdot 2) \dots (2 \cdot 16)(1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{16} (1 \cdot 2 \cdot 3 \dots 16)(1 \cdot 3 \cdot 5 \dots 33) \dots (i)$$

$$\Rightarrow 33! \text{ is divisible by } 2^{16} \therefore 33! \text{ is divisible by } 2^{15}.$$

Now, from (i), we have

$$33! = 2^{16} (2 \cdot 4 \dots 16)(1 \cdot 3 \cdot 5 \dots 15)(1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{16} \cdot 2^8 (1 \cdot 2 \cdot 3 \dots 8)(1 \cdot 3 \dots 15)(1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{24} (2 \cdot 4 \cdot 6 \cdot 8) (1 \cdot 3 \cdot 5 \cdot 7) (1 \cdot 3 \dots 15) (1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{28} (1 \cdot 2 \cdot 3 \cdot 4)(1 \cdot 3 \cdot 5 \cdot 7)(1 \cdot 3 \dots 15)(1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{28} (2 \cdot 4)(1 \cdot 3)(1 \cdot 3 \cdot 5 \cdot 7)(1 \cdot 3 \dots 15)(1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{28} \cdot 2^3 \cdot (1 \cdot 3) \cdot (1 \cdot 3 \cdot 5 \cdot 7) \cdot (1 \cdot 3 \dots 15)(1 \cdot 3 \cdot 5 \dots 33)$$

$$= 2^{31} \times \text{Some odd number.} \dots (ii)$$

From (ii), it is clear that the largest value of n for which $33!$ is divisible by 2^n is 31.

$$33. \quad \text{Clearly } (\sqrt{3} + \sqrt{2})^6 = {}^6C_0(\sqrt{3})^6 + {}^6C_1(\sqrt{3})^5(\sqrt{2})^1$$

$$+ {}^6C_2(\sqrt{3})^4(\sqrt{2})^2 + {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 + {}^6C_4(\sqrt{3})^2(\sqrt{2})^4$$

$$+ {}^6C_5(\sqrt{3})^1(\sqrt{2})^5 + {}^6C_6(\sqrt{2})^6 \dots (i)$$

$$\text{and } (\sqrt{3} - \sqrt{2})^6 = {}^6C_0(\sqrt{3})^6 - {}^6C_1(\sqrt{3})^5(\sqrt{2})^1$$

$$+ {}^6C_2(\sqrt{3})^4(\sqrt{2})^2 - {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 + {}^6C_4(\sqrt{3})^2(\sqrt{2})^4$$

$$- {}^6C_5(\sqrt{3})(\sqrt{2})^5 + {}^6C_6(\sqrt{2})^6 \dots (ii)$$

Adding (i) and (ii), we get

$$(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6 = 2[{}^6C_0(\sqrt{3})^6 + {}^6C_2(\sqrt{3})^4(\sqrt{2})^2$$

$$+ {}^6C_4(\sqrt{3})^2(\sqrt{2})^4 + {}^6C_6(\sqrt{2})^6]$$

$$= 2 \left[1 \cdot 3^3 + \frac{6 \times 5}{2 \times 1} \cdot 3^2 \cdot 2 + \frac{6 \times 5}{2 \times 1} \cdot 3 \cdot 2^2 + 1 \cdot 2^3 \right]$$

$$= 2[27 + 270 + 180 + 8] = 2(485) = 970$$

OR

Consider $8^{2n} - (62)^{2n+1} = 64^n - (62)^{2n+1}$

$$= (1 + 63)^n - (63 - 1)^{2n+1}$$

$$= (1 + 63)^n + (1 - 63)^{2n+1} [\because (1 - 63)^{2n+1} = -(63 - 1)^{2n+1}]$$

$$= [1 + {}^nC_1(63) + {}^nC_2(63)^2 + \dots + {}^nC_n(63)^n]$$

$$+ [1 - {}^{2n+1}C_1(63) + {}^{2n+1}C_2(63)^2 - {}^{2n+1}C_3(63)^3 + \dots$$

$$- {}^{2n+1}C_{2n+1}(63)^{2n+1}]$$

$$= 2 + 63[({}^nC_1 + {}^nC_2(63) + \dots + (63)^{n-1}) + (-{}^{2n+1}C_1$$

$$+ {}^{2n+1}C_2(63) - \dots - (63)^{2n})]$$

As $63[({}^nC_1 + {}^nC_2(63) + \dots + (63)^{n-1}) + (-{}^{2n+1}C_1$
 $+ {}^{2n+1}C_2(63) - \dots - (63)^{2n})]$ is divisible by 9
 $\therefore$ Required remainder is 2.

34. The given inequations are

$$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \text{ and } \frac{7x-1}{3} - \frac{7x+2}{6} > x$$

$$\Rightarrow \frac{4x}{3} - \frac{9}{4} - \frac{3}{4} < x \text{ and } \frac{2(7x-1) - 7x-2}{6} > x$$

$$\Rightarrow \frac{16x-27-9}{12} < x \text{ and } 14x-2-7x-2 > 6x$$

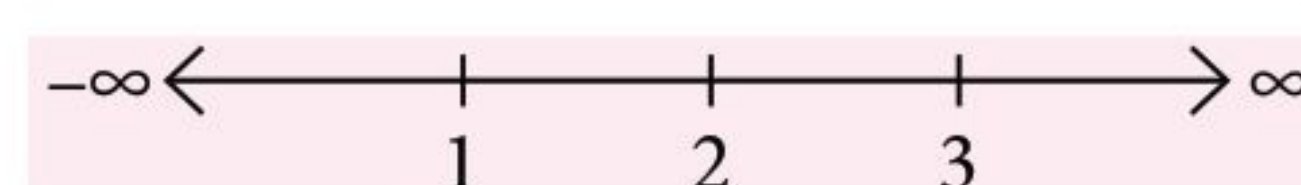
$$\Rightarrow 16x-36 < 12x \text{ and } 7x-6x > 4$$

$$\Rightarrow x < 9 \text{ and } x > 4$$

From above, we have $x \in (4, 9)$.

OR

The given inequation is $|x-1| + |x-2| + |x-3| \leq 6$



Case I : When $-\infty < x \leq 1$

$$\therefore -(x-1) - (x-2) - (x-3) \leq 6 \Rightarrow -3x + 6 \leq 6$$

$$\Rightarrow 3x \geq 0 \Rightarrow x \geq 0$$

But here $x \leq 1 \therefore$ Common value is $0 \leq x \leq 1 \dots (i)$

Case II : When $1 < x \leq 2$

$$\therefore (x-1) - (x-2) - (x-3) \leq 6 \Rightarrow -x + 4 \leq 6$$

$$\Rightarrow -x \leq 2 \Rightarrow x \geq -2.$$

So, common value is $1 < x \leq 2. \dots (ii)$

Case III : When $2 < x \leq 3$

$$\therefore (x-1) + (x-2) - (x-3) \leq 6 \Rightarrow x \leq 6$$

So, common value is $2 < x \leq 3 \dots (iii)$

Case IV : When $3 < x < \infty$

$$\therefore (x-1) + (x-2) + (x-3) \leq 6$$

$$\Rightarrow 3x - 6 \leq 6 \Rightarrow 3x \leq 12 \Rightarrow x \leq 4$$

But in this case $3 < x < \infty$

So, common value is $3 < x \leq 4 \dots (iv)$

$\therefore$ From (i), (ii), (iii) & (iv) solution set of given inequation
 $= [0, 1] \cup (1, 2] \cup (2, 3] \cup (3, 4] = [0, 4].$

35. We have, $\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$

$$\Rightarrow \frac{(2n)!}{7!(2n-7)!} \times \frac{4!(n-4)!}{n!} = \frac{24}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)(2n-7)!}{7 \times 6 \times 5 \times 4 \times (2n-7)!} \times \frac{4!(n-4)!}{n(n-1)(n-2)(n-3)(n-4)!} = 24$$

$$\Rightarrow \frac{2(2n-1)2(n-1)(2n-3)2(n-2)(2n-5)2(n-3)}{7 \times 6 \times 5} \times \frac{1}{(n-1)(n-2)(n-3)} = 24$$

$$\Rightarrow \frac{16 \times (2n-1)(2n-3)(2n-5)}{7 \times 6 \times 5} = 24$$

$$\Rightarrow (2n-1)(2n-3)(2n-5) = \frac{24 \times 7 \times 6 \times 5}{16} = \frac{6 \times 7 \times 6 \times 5}{4}$$

$$= 3 \times 3 \times 7 \times 5 = 9 \times 7 \times 5$$

$$\Rightarrow 2n-1 = 9 \Rightarrow 2n = 10 \Rightarrow n = 5.$$

36. (i) It is given that, password consists of 2 digits followed by 2 letters.

Finding possible number of passwords is same as finding ways of filling four vacant places, 2 with numbers & 2 with letters.

$\therefore$ Required number of passwords = $10 \times 10 \times 26 \times 26 = 67600$

(ii) If repetition of numbers and letters is not permitted, then first two places can be filled in 10×9 ways and last two places can be filled in 26×25 ways.

$\therefore$ Total number of possibilities = 58500

(iii) Since, Ramesh remembered the first and last entry of the password. Therefore he need to try for second and third entry only.

$\therefore$ Number of possible ways to enter 2nd and 3rd entry = 9×25 (As repetition is not allowed) = 225.

$\therefore$ Possible number of passwords = 225

OR

(iii) Now, if he remembered first two and last entry, then he need to try only third entry of password and number of ways of doing so will be 26 or 25 depends on words are repeated or not.

37. (i) The expansion of $(x+a)^n$ has $(n+1)$ terms so, the expansion of $(2x-7y)^7$ has $7+1=8$ terms.

(ii) If n is odd, then the expansion of $(x+a)^n + (x-a)^n$ contains $\left(\frac{n+1}{2}\right)$ terms.

So, the expansion of $((4x+y)^9 + (4x-y)^9)$ has $\left(\frac{9+1}{2}\right) = 5$ terms.

(iii) If n is even, then the expansion of $(x+a)^n - (x-a)^n$ has $\left(\frac{n}{2}\right)$ terms.

So, $((\sqrt{2}+1)^{10} - (\sqrt{2}-1)^{10})$ has $\left(\frac{10}{2}\right) = 5$ terms.

OR

(iii) Since, $(1+x^2+2x) = (1+x)^2$

Thus, $(1+x^2+2x)^{20} = (1+x)^{40}$

So, the number of terms will be $40+1=41$.

38. (i) In order to realise some profit $R(x) > C(x)$

i.e., $45x + 1500 > 15x + 3000$

$\Rightarrow 45x - 15x > 3000 - 1500 \Rightarrow 30x > 1500 \Rightarrow x > 50$

$\therefore$ To get some profit number of items produced and sold should be more than 50.

(ii) Given that, $300 < x < 400$

$\Rightarrow 300 \times 45 < 45x < 400 \times 45 \Rightarrow 13500 < 45x < 18000$

$\Rightarrow 13500 + 1500 < 45x + 1500 < 18000 + 1500$

$\Rightarrow 15000 < R(x) < 19500$

$\therefore$ Revenue will be between 15000 and 19500 rupees.



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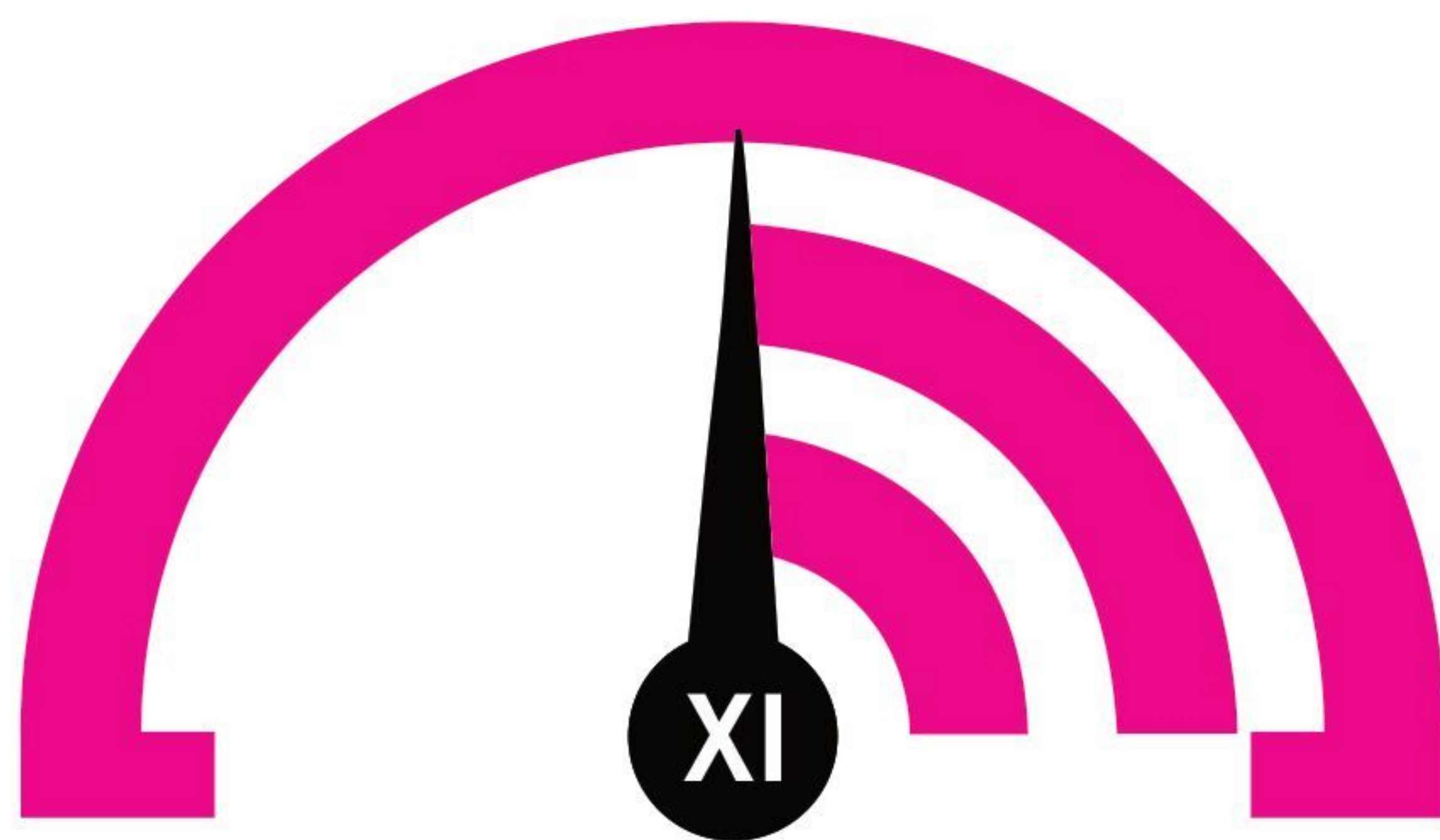
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MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 3 : Trigonometric Functions

Time Taken : 60 Min.

Only One Option Correct Type

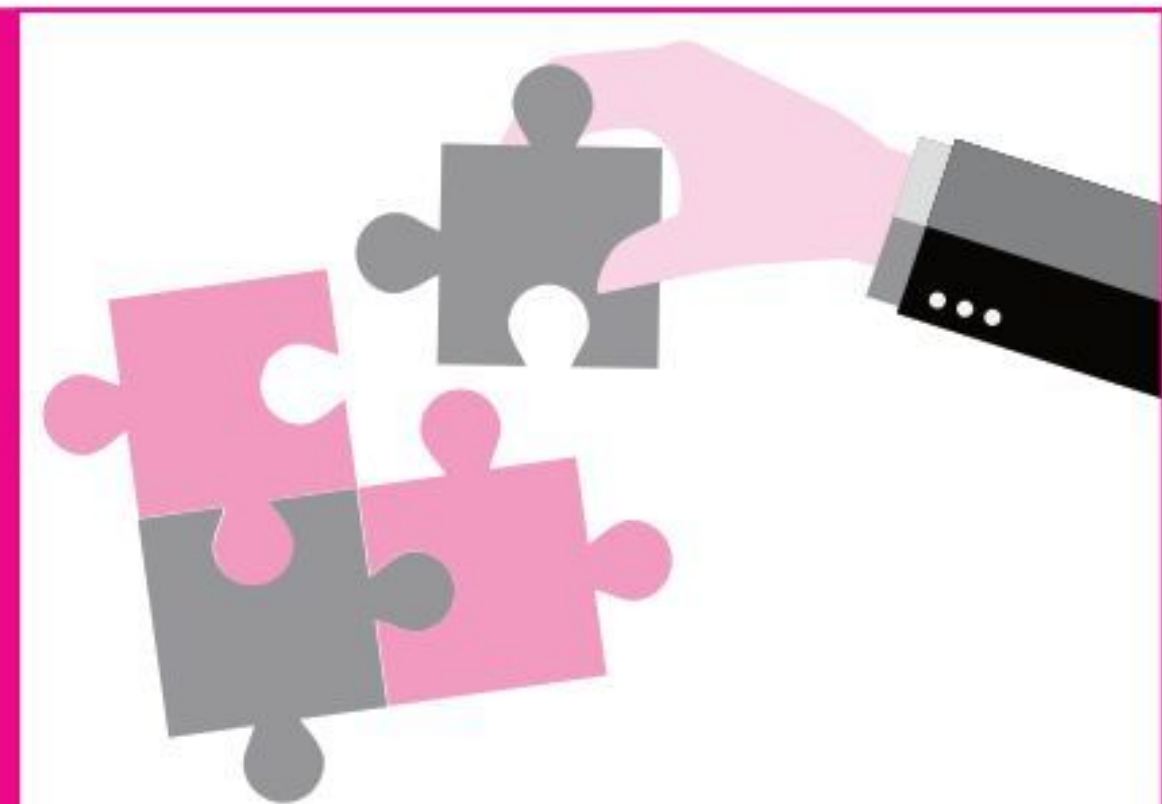
- The maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \cos\left(\frac{\pi}{4} - \theta\right)$ for real values of θ is
(a) 3 (b) 5
(c) 4 (d) none of these
- If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then the general value of θ is
(a) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (b) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$
(c) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (d) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$
- If $\tan \theta \cdot \tan(120^\circ - \theta) \cdot \tan(120^\circ + \theta) = \frac{1}{\sqrt{3}}$, then $\theta =$
(a) $\frac{n\pi}{3} + \frac{\pi}{18}, n \in \mathbb{Z}$ (b) $\frac{n\pi}{3} + \frac{\pi}{12}, n \in \mathbb{Z}$
(c) $\frac{n\pi}{12} + \frac{\pi}{12}, n \in \mathbb{Z}$ (d) $\frac{n\pi}{3} + \frac{\pi}{6}, n \in \mathbb{Z}$
- If $\tan^2 \alpha \tan^2 \beta + \tan^2 \beta \tan^2 \gamma + \tan^2 \gamma \tan^2 \alpha + 2 \tan^2 \alpha \tan^2 \beta \tan^2 \gamma = 1$, then the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is
(a) 0 (b) -1
(c) 1 (d) None of these
- In any triangle, with usual notations the minimum value of $r_1 r_2 r_3 / r_3$ is equal to
(a) 1 (b) 9
(c) 27 (d) None of these
- If $\cos^3 x \sin 2x = \sum_{r=0}^n a_r \sin(rx) \forall x \in \mathbb{R}$, then
(a) $n = 5, a_1 = 1/2$ (b) $n = 5, a_1 = 1/4$
(c) $n = 5, a_2 = 1/8$ (d) $n = 5, a_2 = 1/4$

One or More Than One Option(s) Correct Type

- If $\cos(\theta - \alpha), \cos \theta, \cos(\theta + \alpha)$ are in H.P., then $\cos \theta \sec \frac{\alpha}{2}$ is equal to
(a) -1 (b) $-\sqrt{2}$ (c) $\sqrt{2}$ (d) 2
- Let $f_n(\theta) = \tan \frac{\theta}{2} (1 + \sec \theta)(1 + \sec 2\theta) \dots (1 + \sec 2^{n-1} \theta)$, then
(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$
(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$
- If A is the area and $2s$ is the sum of the sides of a triangle, then

PUZZLE CORNER

ANSWER - SEPTEMBER 2022



7+	15+		9+	2+	
5	6	3	4	1	2
	1-				6+
2	3	6	1	4	5
3+		48×		1-	
3	2	4	6	5	1
	5+		5		3-
1	4	2	5	6	3
2-		25×	12×		
4	1	5	3	2	6
				1-	
6	5	1	2	3	4

- (a) $A \leq \frac{s^2}{4}$ (b) $A \leq \frac{s^2}{3\sqrt{3}}$
 (c) $A > \frac{s^2}{\sqrt{3}}$ (d) None of these

10. If $\sin A = a \cos B$ and $\cos A = b \sin B$, then $(a^2 - 1) \tan^2 A + (1 - b^2) \tan^2 B$ is equal to

- (a) $\frac{a^2 - b^2}{b^2}$ (b) $\frac{a^2 - b^2}{a^2}$
 (c) $\frac{a^2 + b^2}{b^2}$ (d) $\frac{a^2 + b^2}{a^2}$

11. If $\cos^3 x \sin 2x = \sum_{m=1}^n a_m \sin mx$ is an identity in x , then which one of the following is correct?

- (a) $a_2 = 0, a_3 = \frac{3}{8}$ (b) $a_1 = \frac{1}{2}, n = 6$
 (c) $a_1 = \frac{1}{4}, n = 5$ (d) $\sum_{m=1}^n a_m = \frac{3}{4}$

12. If sides of triangle ABC are a, b and c such that $2b = a + c$, then

- (a) $\frac{b}{c} > \frac{2}{3}$ (b) $\frac{b}{c} > \frac{1}{3}$
 (c) $\frac{b}{c} < 2$ (d) $\frac{b}{c} < \frac{1}{2}$

13. If $4\sin_4 x + \cos_4 x = 1$, then x is equal to

- (a) $n\pi, n \in Z$ (b) $n\pi \pm \sin^{-1} \sqrt{\frac{2}{5}}, n \in Z$
 (c) $\frac{2n\pi}{3}, n \in Z$ (d) $2n\pi \pm \frac{\pi}{4}, n \in Z$

Comprehension Type

Paragraph for Q. No. 14 and 15

Consider the system of equations:

$$5^{(\csc^2 x - 3 \sec^2 y)} = 1 \text{ and } 2^{(2 \csc x + \sqrt{3} |\sec y|)} = 64.$$

14. General values of x are

- (a) $n\pi + (-1)^n \pi/6, n \in Z$
 (b) $n\pi + (-1)^n \pi/3, n \in Z$
 (c) $n\pi \pm \frac{\pi}{6}, n \in Z$ (d) $n\pi \pm \frac{\pi}{3}, n \in Z$

15. General values of y are

- (a) $n\pi + (-1)^n \pi/6, n \in Z$
 (b) $n\pi + (-1)^n \pi/3, n \in Z$
 (c) $n\pi \pm \frac{\pi}{6}, n \in Z$ (d) $n\pi \pm \frac{\pi}{3}, n \in Z$

Matrix Match Type

16. Match the following :

Column-I		Column-II	
P.	The value of $\left(\frac{4 + \sec 20^\circ}{\operatorname{cosec} 20^\circ}\right)^2$, is	1.	1
Q.	The minimum value of $\frac{1 + \cos 2x + 8 \sin 2x}{2 \sin 2x}, x \in (0, \pi/2)$, is	2.	2
R.	The value of $\frac{8 \sin 40^\circ \cdot \sin 50^\circ \cdot \tan 10^\circ}{\cos 80^\circ}$ is	3.	3
S.	If $\frac{\cos 5A}{\cos A} + \frac{\sin 5A}{\sin A} = a + b \cos 4A$, then a^2/b is	4.	4

- P Q R S P Q R S
 (a) 1 3 2 4 (b) 3 2 4 1
 (c) 4 1 3 2 (d) 2 4 1 3

Numerical Answer Type

17. Number of solutions of the equation $16(\sin^5 x + \cos^5 x) = 11(\sin x + \cos x)$ in the interval $[0, 2\pi]$ is _____.

18. Number of real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ is _____.

19. If $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3} \sin 250^\circ} = \lambda$, then the value of $9\lambda^4 + 81\lambda^2 + 97$ must be equal to _____.

20. The value of $f(x) = x^4 + 4x^3 + 2x^2 - 4x + 7$, at $x = \cot \frac{11\pi}{8}$, is equal to _____.



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SELF CHECK

No. of questions attempted
 No. of questions correct
 Marks scored in percentage

Check your score! If your score is

- > 90% EXCELLENT WORK ! You are well prepared to take the challenge of final exam.
 90-75% GOOD WORK ! You can score good in the final exam.
 74-60% SATISFACTORY ! You need to score more next time.
 < 60% NOT SATISFACTORY! Revise thoroughly and strengthen your concepts.



Based on CBSE Circular released on 16th Sept. 2022

CBSE warm-up!

CLASS-XII

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Series-3

Continuity and Differentiability/ Application of Derivatives

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If the function $f(x) = \begin{cases} 3x-8, & \text{if } x \leq 5 \\ 2k, & \text{if } x > 5 \end{cases}$ is

continuous, then the value of k is

- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

2. A continuous function is defined as

$$f(x) = \begin{cases} \frac{2x}{\sin x}, & x < 0 \\ \tan ax, & x > 0 \\ x, & x = 0 \end{cases}$$

Then the sum of values of a and b is

- (a) 2 (b) 4 (c) 5 (d) 0

3. If $f(x)$ and $g(x)$ are two continuous functions such that $f(0) = 3$, $g(0) = 2$, then find the value of

$$\lim_{x \rightarrow 0} \{f(x) + g(x)\}.$$

- (a) 2 (b) 3 (c) 5 (d) 7

4. Differentiate $\tan(2x + 3)$ with respect to x .

- (a) $2\sec^2(2x + 3)$ (b) $2\tan^2(2x + 3)$
(c) $2\sec(2x + 3)$ (d) $2\tan(2x + 3)$

5. Find the derivative of $\sin^m \alpha x \cdot \cos^n \beta x$ w.r.t. x .

- (a) $m\sin^{m-1} \alpha x \cdot \alpha + n\cos^{n-1} \beta x \cdot \beta$
(b) $m\alpha(\sin \alpha x)^{m-1} \cos \alpha x \cdot \cos^n \beta x - n\beta \sin^m \alpha x \cdot (\cos \beta x)^{n-1} \sin \beta x$
(c) $m\alpha(\sin \alpha x)^{n-1} \cos \alpha x \cdot \cos^{n-1} \beta x - n\beta \sin^m 2x$
(d) None of these

6. Differentiate $\sqrt{\tan \sqrt{x}}$ w.r.t. x .

- (a) $\frac{\sec^2 x}{4\sqrt{x} \tan \sqrt{x}}$ (b) $\frac{\sec x}{4\sqrt{x} \tan \sqrt{x}}$
(c) $\frac{\sec^2 \sqrt{x}}{4\sqrt{x} \tan \sqrt{x}}$ (d) $\frac{\tan^2 \sqrt{x}}{\sec \sqrt{x}}$

7. If $y = \left\{x + \sqrt{x^2 + a^2}\right\}$, then find $\frac{dy}{dx}$.

- (a) $\frac{\sqrt{x^2 + a^2} - x}{\sqrt{x^2 + a^2}}$ (b) $\frac{x^2 + a^2 - x}{\sqrt{x^2 + a^2}}$

$$(c) \frac{\sqrt{x^2 + a^2} - x^2}{\sqrt{x^2 + a^2}} \quad (d) \frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}}$$

8. Find the derivative of the function f defined by $f(x) = mx + c$ at $x = 0$.

(a) m (b) m^2 (c) $2m$ (d) m^3

9. If $y^2(2 - x) = x^3$, then find $\left(\frac{dy}{dx}\right)_{(1,1)}$.

(a) 2 (b) 4 (c) 6 (d) 8

10. Differentiate $x \cot^8 x$ w.r.t x .

(a) $\cot^7 x (\cot x - 8x)$
 (b) $\cot x (\cot^7 x - 8x \operatorname{cosec}^2 x)$
 (c) $\cot^7 x (\cot x - 8x \operatorname{cosec}^2 x)$
 (d) $\cot x - 8x \operatorname{cosec}^2 x$

11. If $V = \frac{4}{3}\pi r^3$, then the rate at which (in cubic units) V is increasing when $r = 10$ and $\frac{dr}{dt} = 0.01$ is:

(a) π (b) 2π (c) 3π (d) 4π

12. Find the derivative of $2^{\cos^2 x}$ is $-k^{\cos^2 x} \log 2 \sin 2x$, then find the value of k .

(a) 2 (b) 4 (c) 6 (d) 8

13. If x is real, then find the minimum value of $x^2 - 8x + 17$.

(a) 0 (b) -1 (c) 1 (d) 2

14. If $x + y = 8$, then find the maximum value of xy .

(a) 4 (b) 8 (c) 16 (d) 24

15. Find the set of values of ' b ' for which $f(x) = b(x + \cos x) + 4$ is decreasing on R .

(a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(0, 1)$ (d) $(-\infty, 0)$

16. If $f(x)$ attains a local minimum at $x = c$, then write the values of $f'(c)$ and $f''(c)$.

(a) $f'(c) = 0, f''(c) > 0$ (b) $f'(c) > 0, f''(c) = 0$
 (c) $f'(c) < 0, f''(c) = 0$ (d) $f'(c) = 0, f''(c) = 0$

17. Find the value of b for which the function $f(x) = \sin x - bx + c$ is a decreasing function on R .

(a) $b \geq 2$ (b) $b \geq 1$ (c) $b \leq 1$ (d) $b \leq 2$

18. The radius of a circle is increasing at the rate of 0.5 cm/sec. Find the rate of increase (in cm/sec) of its circumference.

(a) 1 (b) 2π (c) 3π (d) π

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.

- (b) Both A and R are true but R is not the correct explanation of A.

- (c) A is true but R is false.

- (d) A is false but R is true.

19. Assertion (A) : If $f(x) = \sqrt{x^2 + 9}$, then the value of $\lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x - 4}$ is $\frac{4}{5}$.

Reason (R) : Let $y = t^{10} + 1$ and $x = t^8 + 1$, then the value of $\frac{d^2 y}{dx^2}$ is $\frac{5}{6t}$.

20. Assertion (A) : The interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing is $(-\infty, -2)$.

Reason (R) : The value of x for which $(x - x^2)$ is maximum is $1/2$.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the derivative of e^{x^3} with respect to $\log x$.

22. If $y = e^{3x+7}$, then find the value of $\left[\frac{dy}{dx}\right]_{x=0}$.

OR

Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

23. Find the interval in which the function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing.

24. Find all the points of local maxima and local minima of the function $f(x) = (x-1)^3(x+1)^2$.

25. If $y = (\tan x)^{\sin x}$, then find dy/dx .

OR

If $f(x) = -\sqrt{25-x^2}$, then find the value of $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$.

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $y = 2\cos(\log x) + 3\sin(\log x)$, then prove that

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$$

27. Find $\frac{dy}{dx}$, when $x = \frac{1 + \log t}{t^2}$, $y = \frac{3 + 2 \log t}{t}$.

OR

If $y = \tan x + \sec x$, prove that $\frac{d^2 y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$.

28. If $y = \sin^{-1} x$, show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 0$.

OR

Differentiate $x^{\sin^{-1} x}$ with respect to $\sin^{-1} x$.

29. Show that $f(x) = \cos(2x + \pi/4)$ is an increasing function on $(3\pi/8, 7\pi/8)$.

OR

Find the shortest distance between the line $y - x = 1$ and the curve $x = y^2$.

30. Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right]$ with respect to $\cos^{-1} x^2$.

31. Show that for $a \geq 1$, $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is decreasing in R .

SECTION - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. The function $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 + ax + b & , 0 \leq x < 2 \\ 3x + 2 & , 2 \leq x \leq 4 \\ 2ax + 5b & , 4 < x \leq 8 \end{cases}$$

If f is continuous on $[0, 8]$, then find the values of a and b .

33. If $y = \frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}$, then show that

$$\frac{dy}{dx} = -\frac{2a^2}{x^3} \left\{ 1 + \frac{a^2}{\sqrt{a^4 - x^4}} \right\}.$$

OR

Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$,

$-1 < x < 1$, $x \neq 0$ w.r.t. x .

34. Find the intervals in which the function given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is
(a) increasing (b) decreasing.

OR

A metal box with a square base and vertical sides is to contain 1024 cm^3 . The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2 . Then, find the least cost of the box.

35. Show that the function $f(x) = |x - 3|$, $x \in R$, is continuous but not differentiable at $x = 3$.

SECTION - E

This section comprises of 3 case-study/passage-based question of 4 marks each with two sub-parts. First two case study question have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case study 1:** Read the following passage and answer the questions given below:

An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 32000 l of water, that comes out from a tube well.



- If the length, width and height of the open tank be x , x and y m respectively, then find the total surface area of tank.
- Find the relation between x and y .
- If the outer surface area of tank will be minimum, then find the depth of tank.

OR

- If cost of aluminium sheet is ₹ 360/ m^2 , then find the minimum cost for the construction of tank.

- 37. Case study 2:** Read the following passage and answer the questions given below:

A function $f(x)$ is said to be continuous in an open interval (a, b) , if it is continuous at every point in this interval.

A function $f(x)$ is said to be continuous in the closed interval $[a, b]$, if $f(x)$ is continuous in (a, b) and $\lim_{h \rightarrow 0} f(a+h) = f(a)$ and $\lim_{h \rightarrow 0} f(b-h) = f(b)$

$$\text{If function } f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & x > 0 \end{cases}$$

is continuous at $x = 0$.

- (i) Find the value of a .
- (ii) Find the value of b .
- (iii) Find the value of c .

OR

- (iii) Find the value of $a + c$.

- 38. Case study 3:** Read the following passage and answer the questions given below:

The Government declare that farmers can get ₹ 200 per quintal for their potatoes on 1st February and after that, the price will be dropped by ₹ 2 per quintal per extra day. Ramu's father has 80 quintal of potatoes in the field and he estimates that crop is increasing at the rate of 1 quintal per day.



- (i) If x is the number of days after 1st February, then express the price and quantity of potatoes respectively in terms of x .
- (ii) Express revenue R as a function of x .

SOLUTIONS

- 1. (d) :** Since $f(x)$ is continuous at $x = 5$,

$$\Rightarrow \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = f(5)$$

$$\Rightarrow 3(5) - 8 = 2k \Rightarrow 7 = 2k \Rightarrow k = \frac{7}{2}$$

- 2. (b) :** Since $f(x)$ is continuous at $x = 0$

$$\text{So, } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} \frac{2x}{\sin x} = \lim_{x \rightarrow 0^+} \frac{\tan ax}{x} = b$$

$$\Rightarrow 2 = a = b$$

$$\textbf{3. (c) : } \lim_{x \rightarrow 0} \{f(x) + g(x)\} = \lim_{x \rightarrow 0} f(x) + \lim_{x \rightarrow 0} g(x)$$

$$= f(0) + g(0)$$

$$= 3 + 2 = 5$$

- 4. (a) :** Let $y = \tan(2x + 3)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \{\tan(2x + 3)\}$$

$$= \sec^2(2x + 3) \cdot 2 = 2 \sec^2(2x + 3)$$

- 5. (b) :** Let $y = \sin^m \alpha x \cdot \cos^n \beta x$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} [\sin^m \alpha x \cdot \cos^n \beta x]$$

$$= \sin^m \alpha x \cdot n(\cos \beta x)^{n-1} (-\sin \beta x \cdot \beta) + \cos^n \beta x \cdot m(\sin \alpha x)^{m-1} (\cos \alpha x \cdot \alpha)$$

- 6. (c) :** Let $y = \sqrt{\tan \sqrt{x}}$

$$\therefore \frac{dy}{dx} = \frac{1}{2} (\tan \sqrt{x})^{-1/2} \cdot \frac{d}{dx} (\tan \sqrt{x})$$

$$= \frac{1}{2\sqrt{\tan \sqrt{x}}} \sec^2 \sqrt{x} \cdot \frac{d}{dx} \sqrt{x}$$

$$= \frac{1}{2\sqrt{\tan \sqrt{x}}} (\sec^2 \sqrt{x}) \frac{1}{2\sqrt{x}} = \frac{\sec^2 \sqrt{x}}{4\sqrt{x}\sqrt{\tan \sqrt{x}}}$$

- 7. (d) :** We have, $y = \left\{x + \sqrt{x^2 + a^2}\right\}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} \left\{x + \sqrt{x^2 + a^2}\right\}$$

$$= 1 + \frac{1}{2\sqrt{x^2 + a^2}} \cdot (2x) = \frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}}$$

- 8. (a) :** Since $f(x)$, being a polynomial, is differentiable everywhere.

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{m(x+h) + c - mx - c}{h}$$

$$= \lim_{h \rightarrow 0} \frac{mh}{h} = m \lim_{h \rightarrow 0} \frac{h}{h} = m$$

9. (a) : Given, $y^2(2-x) = x^3$

$$\Rightarrow y^2 = \frac{x^3}{2-x} \Rightarrow 2y \cdot \frac{dy}{dx} = \frac{(2-x) \times 3x^2 - x^3(-1)}{(2-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x^2 - 2x^3}{2y(2-x)^2} \Rightarrow \left(\frac{dy}{dx} \right)_{(1,1)} = \frac{6-2}{2 \times 1} = 2$$

10. (c) : Let $y = x \cot^8 x$

$$\begin{aligned} \therefore \frac{dy}{dx} &= x \frac{d}{dx} \cot^8 x + \cot^8 x \cdot \frac{dx}{dx} \\ &= x \cdot 8 \cot^7 x (-\operatorname{cosec}^2 x) + \cot^8 x \\ &= \cot^7 x (\cot x - 8x \operatorname{cosec}^2 x). \end{aligned}$$

11. (d) : Given, $V = \frac{4}{3} \pi r^3$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3} \times 3\pi r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\begin{aligned} \Rightarrow \frac{dV}{dt} &= 4\pi(10)^2 \times 0.01 \quad [\text{Given, } r = 10, \frac{dr}{dt} = 0.01] \\ &= 4\pi \times 100 \times 0.01 = 4\pi \quad \therefore k = 2 \end{aligned}$$

12. (a) : Let $y = 2^{\cos^2 x}$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{d}{dx} \left(2^{\cos^2 x} \right) = 2^{\cos^2 x} \log 2 \frac{d}{dx} (\cos^2 x) \\ &= 2^{\cos^2 x} \log 2 [2 \cos x (-\sin x)] \\ &= -2^{\cos^2 x} \log 2 \cdot \sin 2x \end{aligned}$$

13. (c) : Let $f(x) = x^2 - 8x + 17$

$$\therefore f'(x) = 2x - 8$$

$$\text{Now, } f'(x) = 0 \Rightarrow x = 4$$

$$\text{Also, } f''(x) = 2 > 0, \forall x$$

Therefore, $x = 4$ is the point of local minima.

Hence, minimum value of $f(x)$ at $x = 4$ is

$$f(4) = (4 \times 4) - (8 \times 4) + 17 = 1$$

14. (c) : Given $x + y = 8 \Rightarrow y = 8 - x$

$$\text{Let } f(x) = xy = x(8 - x) = 8x - x^2$$

$$\Rightarrow f'(x) = 8 - 2x$$

$$\text{For local maxima or minima } f'(x) = 0$$

$$\Rightarrow 2x = 8 \Rightarrow x = 4 \Rightarrow y = 8 - 4 = 4$$

$$\text{Now, } f''(x) = -2 < 0$$

So, $x = 4$ is a local maxima.

$$\text{Hence, maximum value} = f(4) = 4 \times 4 = 16$$

15. (d) : Here, $f(x) = b(x + \cos x) + 4$

$$\Rightarrow f'(x) = b(1 - \sin x) < 0 \quad [\because f(x) \text{ is decreasing on } R]$$

$$\Rightarrow \text{Now, } \sin x \leq 1 \quad \forall x \in R$$

$$\Rightarrow 1 - \sin x \geq 0 \quad \forall x \in R$$

$\therefore b$ should be negative. Thus, $b \in (-\infty, 0)$.

16. (a) : If $f(x)$ attains a local minimum at $x = c$, then $f'(c) = 0$ and $f''(c) > 0$.

17. (b) : Here, $f(x) = \sin x - bx + c$

$$\therefore f'(x) = \cos x - b$$

Now, $f(x)$ is decreasing on R if $f'(x) \leq 0$, for all $x \in R$

$$\Rightarrow \cos x - b \leq 0, x \in R \Rightarrow \cos x \leq b, x \in R$$

$$\Rightarrow b \geq 1$$

18. (d) : Let r be the radius and C be the circumference of the circle at any time t .

$$\therefore C = 2\pi r \Rightarrow \frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi \times 0.5$$

$$[\because \frac{dr}{dt} = 0.5 \text{ cm/sec}]$$

$$\Rightarrow \frac{dC}{dt} = \pi \text{ cm/sec}$$

19. (c) : Assertion (A) :

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x - 4} &= \lim_{x \rightarrow 4} \frac{\sqrt{x^2 + 9} - 5}{x - 4} \times \frac{\sqrt{x^2 + 9} + 5}{\sqrt{x^2 + 9} + 5} \\ &= \frac{x^2 - 16}{(x - 4)(\sqrt{x^2 + 9} + 5)} = \frac{x + 4}{(\sqrt{x^2 + 9} + 5)} \\ &= \frac{8}{10} = \frac{4}{5} \end{aligned}$$

Reason (R) : We have, $y = t^{10} + 1, x = t^8 + 1$

$$\Rightarrow \frac{dy}{dt} = 10t^9, \frac{dx}{dt} = 8t^7 \therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{10t^9}{8t^7} = \frac{5}{4}t^2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{5}{4}(2t) \frac{dt}{dx} = \frac{5}{4} \times 2t \times \frac{1}{8t^7} = \frac{5}{16t^6}.$$

So, A is true but R is false

20. (b) : Assertion (A) : We have, $f(x) = 7 - 4x - x^2$

$$\therefore f'(x) = -4 - 2x$$

For strictly increasing, $f'(x) > 0$

$$\Rightarrow -4 - 2x > 0 \Rightarrow x < -2$$

$\therefore$ Required interval is $(-\infty, -2)$.

Reason (R) : Let $f(x) = x - x^2$

$$\therefore f'(x) = 1 - 2x$$

For critical point, $f'(x) = 0$

$$\Rightarrow 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$$

$$\text{Now, at } x = \frac{1}{2}, f''(x) = -2 < 0$$

So, $f(x)$ has maximum value at $x = \frac{1}{2}$.

So, both A and R are true but R is not the correct explanation of A.

21. Let $y = e^{x^3}, z = \log x$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = e^{x^3} (3x^2) = 3x^2 e^{x^3} \text{ and } \frac{dz}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3x^2 e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3 e^{x^3}.$$

$$22. \because y = e^{3x+7} \Rightarrow \frac{dy}{dx} = 3e^{3x+7}$$

$$\therefore \left[\frac{dy}{dx} \right]_{x=0} = 3e^{3 \times 0 + 7} = 3e^7$$

OR

$$\text{Let } u = \sin^{-1} \left(\frac{2x}{1+x^2} \right) = 2 \tan^{-1} x$$

$$\text{and } v = \tan^{-1} \left(\frac{2x}{1-x^2} \right) = 2 \tan^{-1} x$$

$$\Rightarrow u = v \Rightarrow \frac{du}{dx} = \frac{dv}{dx} \quad \therefore \frac{du}{dv} = \frac{(du/dx)}{(dv/dx)} = 1$$

$$23. \text{ Given, } f(x) = \log(1+x) - \frac{2x}{2+x}$$

$$\Rightarrow f'(x) = \frac{1}{1+x} - \frac{(2+x)2-2x}{(2+x)^2} \Rightarrow f'(x) = \frac{x^2}{(x+1)(x+2)^2}$$

Clearly, $f'(x) > 0$ for all $x > -1$, hence $f(x)$ is increasing on $(-1, \infty)$.

$$24. \text{ Let } y = f(x) = (x-1)^3(x+1)^2$$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= 3(x-1)^2(x+1)^2 + 2(x+1)(x-1)^3 \\ &= (x-1)^2(x+1)\{3(x+1) + 2(x-1)\} \\ &= (x-1)^2(x+1)(5x+1) \end{aligned}$$

For local maxima or local minima, we have

$$\frac{dy}{dx} = 0 \Rightarrow (x-1)^2(x+1)(5x+1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -1 \text{ or } x = -\frac{1}{5}.$$

$x = -1$ is the point of maxima & $x = -\frac{1}{5}$ is the point of minima.

$$25. \text{ We have, } y = (\tan x)^{\sin x}$$

Taking log on both sides, we get

$$\log y = \sin x \log(\tan x)$$

Differentiating w.r.t. x , we get

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{\sin x}{\tan x} \cdot \sec^2 x + \cos x \log(\tan x) \\ &= (\tan x)^{\sin x} [\sec x + \cos x (\log \tan x)]. \end{aligned}$$

OR

$$\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = f'(1), \text{ Now, } f'(x) = -\frac{1}{2} \cdot \frac{-2x}{\sqrt{25-x^2}}$$

$$\therefore f'(1) = -\frac{1}{2} \cdot \frac{-2 \times 1}{\sqrt{25-1^2}} = \frac{1}{\sqrt{24}}.$$

$$26. \text{ We have, } y = 2 \cos(\log x) + 3 \sin(\log x)$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = -2 \sin(\log x) \times \frac{1}{x} + 3 \cos(\log x) \times \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = -2 \sin(\log x) + 3 \cos(\log x)$$

Again differentiating w.r.t. x , we get

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = -2 \cos(\log x) \times \frac{1}{x} - 3 \sin(\log x) \times \frac{1}{x}$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -[2 \cos(\log x) + 3 \sin(\log x)]$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -y \Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$27. \text{ We have, } x = \frac{1 + \log t}{t^2}$$

$$\text{and } y = \frac{3 + 2 \log t}{t}$$

$$\frac{dx}{dt} = \frac{t^2 \cdot \frac{d}{dt}(1 + \log t) - (1 + \log t) \cdot \frac{d}{dt} t^2}{(t^2)^2}$$

$$= \frac{t^2 \cdot \frac{1}{t} - (1 + \log t) \cdot 2t}{t^4} = \frac{t - (1 + \log t) \cdot 2t}{t^4}$$

$$= \frac{t}{t^4} [1 - 2(1 + \log t)] = \frac{-1 - 2 \log t}{t^3} \quad \dots(i)$$

$$\text{and } \frac{dy}{dt} = \frac{t \cdot \frac{d}{dt}(3 + 2 \log t) - (3 + 2 \log t) \cdot \frac{d}{dt} t}{t^2}$$

$$= \frac{t \cdot 2 \cdot \frac{1}{t} - (3 + 2 \log t) \cdot 1}{t^2} = \frac{2 - 3 - 2 \log t}{t^2} = \frac{-1 - 2 \log t}{t^2} \quad \dots(ii)$$

Dividing (ii) by (i), we get

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(-1 - 2 \log t)/t^2}{(-1 - 2 \log t)/t^3} = t$$

OR

Here, $y = \tan x + \sec x$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x = \frac{1}{\cos^2 x} + \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos^2 x} = \frac{1 + \sin x}{1 - \sin^2 x} = \frac{1 + \sin x}{(1 + \sin x)(1 - \sin x)}$$

$$= \frac{1}{1 - \sin x} = (1 - \sin x)^{-1}$$

Again differentiating w.r.t. x , we have

$$\frac{d^2 y}{dx^2} = (-1)(1 - \sin x)^{-2} \frac{d}{dx}(1 - \sin x)$$

$$= \frac{-1}{(1 - \sin x)^2} (-\cos x) = \frac{\cos x}{(1 - \sin x)^2}$$

28. Given $y = \sin^{-1} x$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Again differentiating w.r.t. x , we get

$$\frac{d^2 y}{dx^2} = \frac{\sqrt{1-x^2} \cdot 0 - 1 \cdot \frac{1(-2x)}{2\sqrt{1-x^2}}}{1-x^2} \Rightarrow (1-x^2) \frac{d^2 y}{dx^2} = \frac{x}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - \frac{x}{\sqrt{1-x^2}} = 0 \Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - x \cdot \frac{dy}{dx} = 0$$

OR

Let $u = x^{\sin^{-1} x}$ and $v = \sin^{-1} x$. Then,

$$u = x^{\sin^{-1} x} \Rightarrow u = e^{\sin^{-1} x \cdot \log x}$$

$$\Rightarrow \frac{du}{dx} = e^{\sin^{-1} x \cdot \log x} \cdot \frac{d}{dx} \left\{ \sin^{-1} x \cdot \log x \right\}$$

$$\Rightarrow \frac{du}{dx} = x^{\sin^{-1} x} \left\{ \frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right\}$$

$$\text{and } v = \sin^{-1} x \Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{x^{\sin^{-1} x} \left\{ \frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right\}}{\frac{1}{\sqrt{1-x^2}}}$$

$$= x^{\sin^{-1} x} \left\{ \log x + \frac{\sqrt{1-x^2}}{x} \sin^{-1} x \right\}$$

29. We have, $f(x) = \cos(2x + \pi/4)$

$$\therefore f'(x) = -2\sin(2x + \pi/4)$$

$$\text{Now, } x \in (3\pi/8, 7\pi/8) \Rightarrow 3\pi/8 < x < 7\pi/8$$

$$\Rightarrow 3\pi/4 < 2x < 7\pi/4$$

$$\Rightarrow \pi/4 + 3\pi/4 < 2x + \pi/4 < 7\pi/4 + \pi/4$$

$$\Rightarrow \pi < 2x + \pi/4 < 2\pi$$

$$\Rightarrow \sin(2x + \pi/4) < 0$$

[$\because$ sine function is negative in third and fourth quadrants]

$$\Rightarrow -2\sin(2x + \pi/4) > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is increasing on $(3\pi/8, 7\pi/8)$.

OR

Let $P(t^2, t)$ be any point on the curve $x = y^2$. The distance S of P from the given line is

$$S = \left| \frac{t - t^2 - 1}{\sqrt{2}} \right| = \left| \frac{t^2 - t + 1}{\sqrt{2}} \right| = \frac{t^2 - t + 1}{\sqrt{2}}$$

[$\because t^2 - t + 1 > 0$ for all $t \in \mathbb{R}$]

$$\therefore \frac{dS}{dt} = \frac{2t-1}{\sqrt{2}}$$

For the critical points of S , $\frac{dS}{dt} = 0$.

$$\Rightarrow 2t - 1 = 0 \Rightarrow t = \frac{1}{2}$$

Now, $\frac{d^2 S}{dt^2} = \frac{2}{\sqrt{2}} = \sqrt{2} > 0$, for all t . So, S is minimum

when $t = \frac{1}{2}$.

Putting $t = \frac{1}{2}$ in $S = \frac{t^2 - t + 1}{\sqrt{2}}$, the minimum value of S

$$\text{is given by } S = \frac{\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1}{\sqrt{2}} = \frac{3\sqrt{2}}{8}.$$

30. Let $u = \tan^{-1} \left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right]$... (i)

Putting $x^2 = \cos 2\theta$ in (i), we get

$$u = \tan^{-1} \left[\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right]$$

$$= \tan^{-1} \left[\frac{\sqrt{2\cos^2 \theta} - \sqrt{2\sin^2 \theta}}{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}} \right]$$

$$= \tan^{-1} \left[\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right] = \tan^{-1} \left[\frac{1 - \tan \theta}{1 + \tan \theta} \right]$$

$$= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \theta \right) \right] = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{\cos^{-1}(x^2)}{2}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2} \frac{1}{\sqrt{1-x^4}} 2x = \frac{x}{\sqrt{1-x^4}} \quad \dots(ii)$$

$$\text{Let } v = \cos^{-1}(x^2) \Rightarrow \frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}} \quad \dots(iii)$$

$$\text{Dividing (ii) by (iii), we get } \frac{du}{dv} = -\frac{1}{2}.$$

31. Given that, $a \geq 1$,

$$f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$$

$$\Rightarrow f'(x) = \sqrt{3} \cos x - (-\sin x) - 2a$$

$$= \sqrt{3} \cos x + \sin x - 2a$$

$$= 2 \left[\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right] - 2a$$

$$= 2 \left[\cos \frac{\pi}{6} \cos x + \sin \frac{\pi}{6} \sin x \right] - 2a = 2 \cos \left(\frac{\pi}{6} - x \right) - 2a$$

$$[\because \cos(A - B) = \cos A \cos B + \sin A \sin B]$$

$$= 2 \left[\cos \left(\frac{\pi}{6} - x \right) - a \right]$$

Since, $\cos x \in [-1, 1]$ and $a \geq 1$

$$\text{So, } 2 \left[\cos \left(\frac{\pi}{6} - x \right) - a \right] \leq 0 \Rightarrow f'(x) \leq 0$$

Therefore, $f(x)$ is a decreasing function in R .

Hence proved.

$$\text{32. Here, } f(x) = \begin{cases} x^2 + ax + b, & 0 \leq x < 2 \\ 3x + 2, & 2 \leq x \leq 4 \\ 2ax + 5b, & 4 < x \leq 8 \end{cases}$$

Since, it is given that f is continuous on $[0, 8]$ i.e., $f(x)$ is continuous at $x = 2$ and $x = 4$.

$$\therefore \text{L.H.L.} = f(x) = \text{R.H.L.}$$

$$\text{At } x = 2: \lim_{x \rightarrow 2^-} f(x) = f(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} (x^2 + ax + b) = 3 \times 2 + 2$$

$$\lim_{h \rightarrow 0} [(2-h)^2 + a(2-h) + b] = 8 \Rightarrow 4 + 2a + b = 8$$

$$\Rightarrow 2a + b = 4 \quad \dots(i)$$

$$\text{At } x = 4: \lim_{x \rightarrow 4^+} f(x) = f(4)$$

$$\Rightarrow \lim_{x \rightarrow 4^+} (2ax + 5b) = 3 \times 4 + 2$$

$$\Rightarrow \lim_{h \rightarrow 0} [2a(4+h) + 5b] = 14 \Rightarrow 8a + 5b = 14 \quad \dots(ii)$$

Solving (i) and (ii), we get $a = 3, b = -2$.

$$\text{33. We have, } y = \frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}$$

$$= \frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}} \times \frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}$$

$$\Rightarrow y = \frac{\left\{ \sqrt{a^2 + x^2} + \sqrt{a^2 - x^2} \right\}^2}{(a^2 + x^2) - (a^2 - x^2)}$$

$$\Rightarrow y = \frac{2a^2 + 2\sqrt{a^4 - x^4}}{2x^2} = \frac{a^2}{x^2} + \frac{\sqrt{a^4 - x^4}}{x^2}$$

$$= a^2 x^{-2} + \sqrt{a^4 - x^4} x^{-2}$$

$$\therefore \frac{dy}{dx} = a^2 \frac{d}{dx}(x^{-2}) + \frac{d}{dx} \left\{ \sqrt{a^4 - x^4} x^{-2} \right\}$$

$$= -2a^2 x^{-3} + (-2)x^{-3} \sqrt{a^4 - x^4}$$

$$+ (x^{-2}) \frac{1}{2} (a^4 - x^4)^{-1/2} \times \frac{d}{dx} (a^4 - x^4)$$

$$= -\frac{2a^2}{x^3} - \frac{2}{x^3} \sqrt{a^4 - x^4} + \frac{1}{2x^2 \sqrt{a^4 - x^4}} (-4x^3)$$

$$= -\frac{2a^2}{x^3} - \frac{2}{x^3} \sqrt{a^4 - x^4} - \frac{2x}{\sqrt{a^4 - x^4}}$$

$$= -\frac{2a^2}{x^3} - 2 \left\{ \frac{\sqrt{a^4 - x^4}}{x^3} + \frac{x}{\sqrt{a^4 - x^4}} \right\}$$

$$= -\frac{2a^2}{x^3} - 2 \left\{ \frac{a^4 - x^4 + x^4}{x^3 \sqrt{a^4 - x^4}} \right\}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2a^2}{x^3} - \frac{2a^4}{x^3 \sqrt{a^4 - x^4}} = -\frac{2a^2}{x^3} \left\{ 1 + \frac{a^2}{\sqrt{a^4 - x^4}} \right\}$$

OR

$$\text{Let } y = \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] \quad \dots(i)$$

Substituting $x^2 = \cos 2\theta$ in (i), we get

$$y = \tan^{-1} \left(\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{1+2\cos^2 \theta - 1} + \sqrt{1-1+2\sin^2 \theta}}{\sqrt{1+2\cos^2 \theta - 1} - \sqrt{1-1+2\sin^2 \theta}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{2} \cos \theta + \sqrt{2} \sin \theta}{\sqrt{2} \cos \theta - \sqrt{2} \sin \theta} \right) \left[\because \cos 2x = 2\cos^2 x - 1 \right. \\ \left. = 1 - 2\sin^2 x \right]$$

$$\begin{aligned}
&= \tan^{-1} \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) = \tan^{-1} \left(\frac{1 + \tan \theta}{1 - \tan \theta} \right) \\
&= \tan^{-1} \left(\tan \left(\frac{\pi}{4} + \theta \right) \right) \left[\because \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b} \right] \\
&= \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2 \left[\because x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2} \cos^{-1} x^2 \right] \\
\therefore \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\pi}{4} \right) + \frac{d}{dx} \left(\frac{1}{2} \cos^{-1} x^2 \right) \\
&= 0 + \frac{1}{2} \cdot \frac{-1}{\sqrt{1-x^4}} \cdot \frac{d}{dx} (x^2) = \frac{1}{2} \cdot \frac{-2x}{\sqrt{1-x^4}} = \frac{-x}{\sqrt{1-x^4}}
\end{aligned}$$

34. The given function is $f(x) = \sin x + \cos x$; $0 \leq x \leq 2\pi$
 $\Rightarrow f'(x) = \cos x - \sin x$
Now $f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

$$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Thus $f'(x) > 0$ in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

$$f'(x) < 0 \Rightarrow x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$$

$\Rightarrow$ The function f is decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ and it is increasing in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$.

OR

Let length and height of the square box be x cm and y cm respectively.

Then, volume of the box (V) = $x^2y = 1024$ (given)

$$\text{Since, } x^2y = 1024 \Rightarrow y = \frac{1024}{x^2}$$

If C denotes the cost of the box.

Then, $C = (2x^2 \times 5) + (4xy \times 2.50)$

$$= 10x^2 + 10xy = 10x(x+y) = 10x \left(x + \frac{1024}{x^2} \right)$$

$$\Rightarrow C = 10x^2 + \frac{10240}{x} \quad \dots(i)$$

On differentiating both sides with respect to x , we get

$$\frac{dC}{dx} = 20x - \frac{10240}{x^2} \quad \dots(ii)$$

$$\text{Put } \frac{dC}{dx} = 0 \Rightarrow 20x = \frac{10240}{x^2} \Rightarrow 20x^3 = 10240$$

$$\Rightarrow x^3 = 512 = 8^3 \Rightarrow x = 8$$

Again, differentiating (ii) with respect to x , we get

$$\frac{d^2C}{dx^2} = 20 - 10240(-2) \cdot \frac{1}{x^3} = 20 + \frac{20480}{x^3}$$

$$\therefore \left(\frac{d^2C}{dx^2} \right)_{x=8} = 20 + \frac{20480}{512} = 60 > 0$$

Hence, cost is minimum for $x = 8$ and the corresponding least cost of the box is,

$$C = 10(8)^2 + \frac{10240}{8} \quad (\text{From (i)})$$

$$= 640 + 1280 = ₹ 1920$$

$$\mathbf{35.} \quad f(x) = |x-3| = \begin{cases} x-3, & \text{if } x \geq 3 \\ -(x-3), & \text{if } x < 3 \end{cases}$$

We have, $f(3) = (3-3) = 0$

$$\begin{aligned}
\text{R.H.L.} &= \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) \\
&= \lim_{h \rightarrow 0} (3+h) - 3 = \lim_{h \rightarrow 0} h = 0
\end{aligned}$$

$$\text{L.H.L.} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3-h) = \lim_{h \rightarrow 0} [-(3-h-3)] = \lim_{h \rightarrow 0} h = 0$$

$$\therefore \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^-} f(x) = f(3) = 0$$

So, $f(x)$ is continuous at $x = 3$.

$$\text{Now, } Rf'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(3+h-3) - 0}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

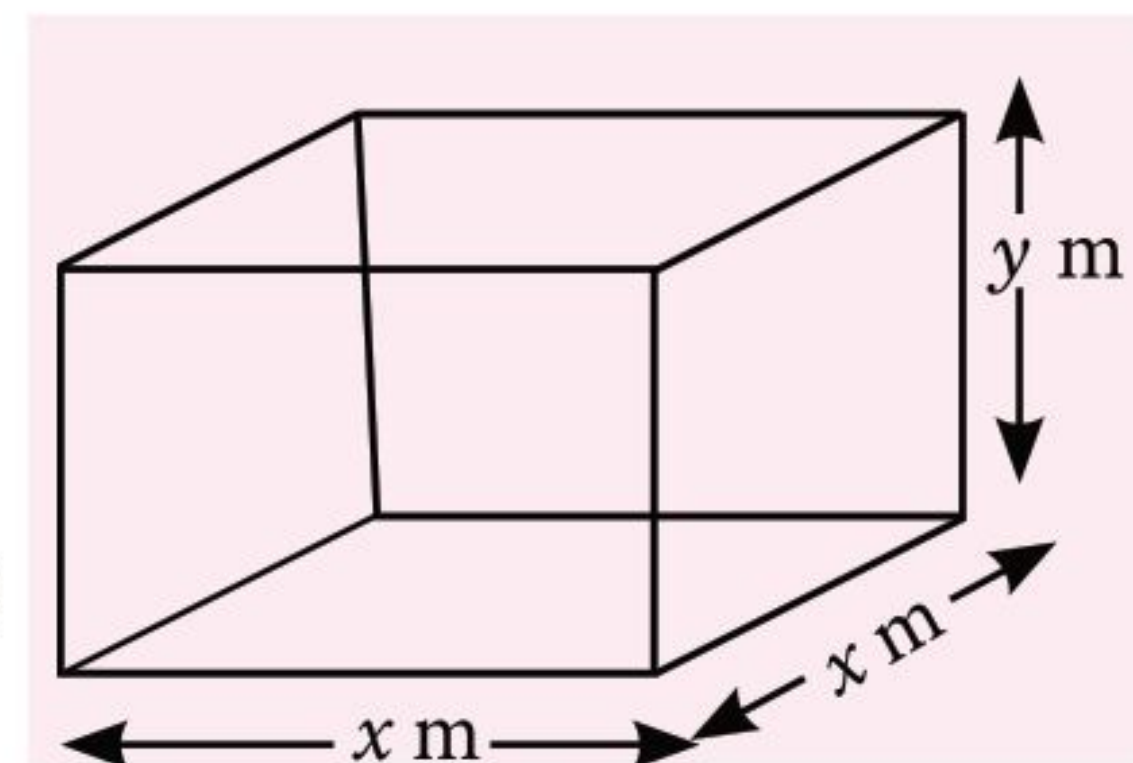
$$\text{And } Lf'(3) = \lim_{h \rightarrow 0} \frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{[-(3-h-3)] - 0}{-h} = \lim_{h \rightarrow 0} \frac{h}{-h} = -1$$

Thus, $Rf'(3) \neq Lf'(3)$

$\therefore f(x)$ is not differentiable at $x = 3$.

36. (i) Since, the tank is open from the top, therefore the total surface area is
= (Outer + Inner) surface area



$$= 2(x \times x + 2(xy + yx)) = 2(x^2 + 2(2xy)) = (2x^2 + 8xy) \text{ m}^2$$

(ii) Since, volume of tank should be 32000 l.

$$\therefore x^2y \text{ m}^3 = 32000 \text{ l} = 32 \text{ m}^3 \quad [\because 1 \text{ litre} = 0.001 \text{ m}^3]$$

So, $x^2y = 32$

(iii) Let S be the outer surface area of tank.

Then, $S = x^2 + 4xy$

$$\Rightarrow S(x) = x^2 + 4x \cdot \frac{32}{x^2} = x^2 + \frac{128}{x} \quad [\because x^2y = 32]$$

$$\Rightarrow \frac{dS}{dx} = 2x - \frac{128}{x^2} \text{ and } \frac{d^2S}{dx^2} = 2 + \frac{256}{x^3}$$

For maximum or minimum values of S , consider $\frac{dS}{dx} = 0$

$$\Rightarrow 2x = \frac{128}{x^2} \Rightarrow x^3 = 64 \Rightarrow x = 4 \text{ m}$$

$$\text{At } x = 4, \frac{d^2S}{dx^2} = 2 + \frac{256}{4^3} = 2 + 4 = 6 > 0$$

$\therefore S$ is minimum when $x = 4$

Now as $x^2y = 32$, therefore $y = 2$. Thus, $x = 2y$.

OR

(iii) Since, minimum surface area

$$= x^2 + 4xy = 4^2 + 4 \times 4 \times 2 = 48 \text{ m}^2 \text{ and}$$

Cost per $\text{m}^2 = ₹ 360$

$\therefore$ Minimum cost is $= ₹ (48 \times 360) = ₹ 17280$

$$37. \text{ L.H.L. (at } x = 0) = \lim_{x \rightarrow 0} \frac{\sin(a+1)x + \sin x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(a+1)\sin(a+1)x}{(a+1)x} + \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$= a + 1 + 1 = a + 2$$

...(1)

$$\text{R.H.L. (at } x = 0) = \lim_{x \rightarrow 0} \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} = \lim_{x \rightarrow 0} \frac{\sqrt{1+bx} - 1}{bx}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+bx} + 1} = \frac{1}{2} \quad \dots(2)$$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore$ From (1) and (2), we get

$$a + 2 = c = \frac{1}{2} \Rightarrow a = -\frac{3}{2}, c = \frac{1}{2}$$

Also, value of b does not affect the continuity of $f(x)$, so b can be any real number.

$$(i) \quad a = -\frac{3}{2}$$

(ii) b can be any real number

$$(iii) \quad c = \frac{1}{2}$$

OR

$$(iii) \quad a + c = -\frac{3}{2} + \frac{1}{2} = -1$$

38. (i) If x be the number of extra days after 1st February

$$\therefore \text{Price} = ₹(200 - 2 \times x) = ₹(200 - 2x)$$

$$\text{Quantity} = 800 \text{ quintals} + x(1 \text{ quintal per day}) \\ = (80 + x) \text{ quintals}$$

(ii) $R(x) = \text{Quantity} \times \text{Price}$

$$= (80 + x)(200 - 2x) = 16000 - 160x + 200x - 2x^2 \\ = 16000 + 40x - 2x^2$$



UNSCRAMBLE ME

Unscramble the words given in column I and match them with their explanations in column II.

Column I

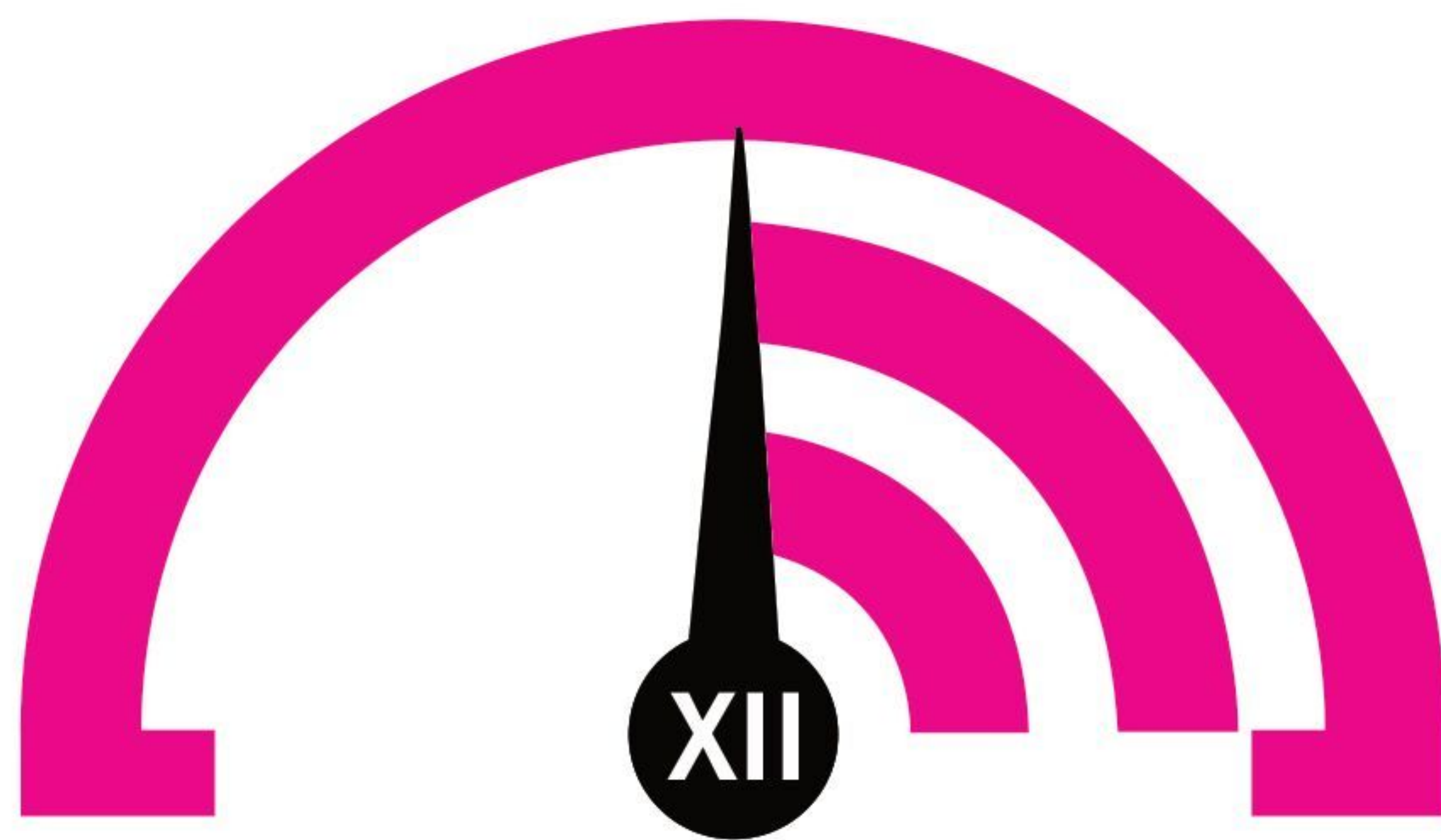
- OMINTCBAINO
- ERATLNOI
- EMTAOPRUTIN
- UCIOFNTN
- ARXMTI
- MGARIAINY
- NQALIEUYT
- ROONTIGEYRMT
- RBIITPOABLY
- ETEGYOMR

Column II

- An arrangement of objects in which order of selection matters.
- A relation in which every input is paired with exactly one output.
- A branch of mathematics that deals with the relationship between sides and angles of a triangle.
- An arrangement of objects in which order of selection does not matter.
- A connection between the elements of two or more non empty sets.
- Rectangular arrangements of number.
- A measure of the likelihood that an event in the future will happen and assume a value between 0 and 1.
- A branch of mathematics concerned with the property and relationship between points, lines, planes and figures.
- A non real number that is a multiple of i , which is the square root of -1 .
- A mathematical sentence that contains the following symbols $<, >, \leq, \geq$.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month.
Winners' names will be published in next issue.

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 3 : Matrices and Determinants

Time Taken : 60 Min.

Only One Option Correct Type

1. If a, b, c are in G.P. or $x - \alpha$ is a factor of $ax^2 + 2bx + c = 0$, then value of the determinant

$$\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} \text{ is}$$

- (a) 0 (b) 1 (c) 2 (d) 3

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 3 & -1 & 9 \end{bmatrix}$, then the value of

$\det(\text{adj}(\text{adj} A))$ equals

- (a) 11 (b) 121 (c) 1331 (d) 14641

3. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$, if

- (a) x, y, z are in A.P. (b) x, y, z are in G.P.
(c) x, y, z are in H.P. (d) xy, yz, zx are in A.P.

4. Let $\Delta(x) = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ (x-1) & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$, then the

coefficient of x in $\Delta(x)$ is

- (a) -3 (b) -2 (c) -1 (d) 0

5. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$.

Then $\text{tr}(A) - \text{tr}(B)$ has the value equal to

- (a) 0 (b) 1 (c) 2 (d) 3

6. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is

- (a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

One or More Than One Option(s) Correct Type

7. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x)$ is

divisible by

- (a) x (b) a (c) $2a + 3x$ (d) x^2

8. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

- (a) $A^2 - 4A - 5I = O$ (b) $A^{-1} = \frac{1}{5}(A - 4I)$
(c) $A^2 - 3A - 3I = O$ (d) $A^{-1} = \frac{1}{3}(A - 3I)$

9. If $A + B + C = \pi$, $e^{i\theta} = \cos\theta + i \sin\theta$ and

$$z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}, \text{ then}$$

- (a) $\text{Re}(z) = 4$ (b) $\text{Im}(z) = 0$
(c) $\text{Re}(z) = -4$ (d) $\text{Im}(z) = -1$

10. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ be a matrix. If $A^{10} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

- (a) number of divisors of a is 11
 (b) a is an odd integer
 (c) $(a + b + d)$ is an even integer
 (d) $a + d$ is a multiple of 13

11. If $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal matrix, then

- (a) $a = \pm \frac{1}{\sqrt{2}}$ (b) $b = \pm \frac{1}{\sqrt{12}}$
 (c) $c = \pm \frac{1}{\sqrt{3}}$ (d) None of these

12. The value of θ for which the system of linear equations in x, y, z given as, $(\sin 3\theta)x - y + z = 0$, $(\cos 2\theta)x + 4y + 3z = 0$, $2x + 7y + 7z = 0$ has a non-trivial solution, is/are

- (a) $\frac{n\pi}{2}$ (b) $\frac{2n\pi}{3}$
 (c) $n\pi + (-1)^n \frac{\pi}{6}$ (d) None of these

13. If A is symmetric and B is skew symmetric matrix, then which of the following is/are not true?

- (a) ABA^T is symmetric matrix
 (b) $AB^T + BA^T$ is symmetric matrix
 (c) $(A + B)(A - B)$ is skew symmetric
 (d) $(A + I)(B - I)$ is skew symmetric

Comprehension Type

Paragraph for Q. No. 14 and 15

Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Also,

$$S_n = \alpha^n + \beta^n \text{ for } n \geq 1 \text{ and } \Delta = \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}.$$

14. If $\Delta < 0$, then the equation $ax^2 + bx + c = 0$ has
 (a) positive real roots (b) negative real roots
 (c) equal roots (d) imaginary roots

15. If a, b, c are rational and one of the roots of the equation is $1 + \sqrt{2}$, then the value of Δ is
 (a) 8 (b) 12 (c) 30 (d) 32

Matrix Match Type

16. If, $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = px^5 + qx^4 + rx^3 + sx^2 + tx + w$, then match the following columns.

Column-I		Column-II	
P.	w is equal to	1.	3
Q.	t is equal to	2.	1
R.	$(p + r)$ is equal to	3.	-1
S.	$(q + s)$ is equal to	4.	0

	P	Q	R	S
(a)	1	2	3	4
(b)	2	1	3	2
(c)	3	1	4	2
(d)	4	2	3	4

Numerical Answer Type

17. If $A^2 = 3A - 2I$ and $A^8 = pA + qI$, then $p + q$ is equal to _____.
18. If A, B and C are $n \times n$ matrices and $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, then the value of $[\det(A^2BC^{-1})]$ (where $[\cdot]$ represents greatest integer function) is _____.

19. The value of $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ is equal to _____.

20. In a ΔABC , if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then the value of $64 (\sin^2 A + \sin^2 B + \sin^2 C)$ must be equal to _____.



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 No. of questions correct
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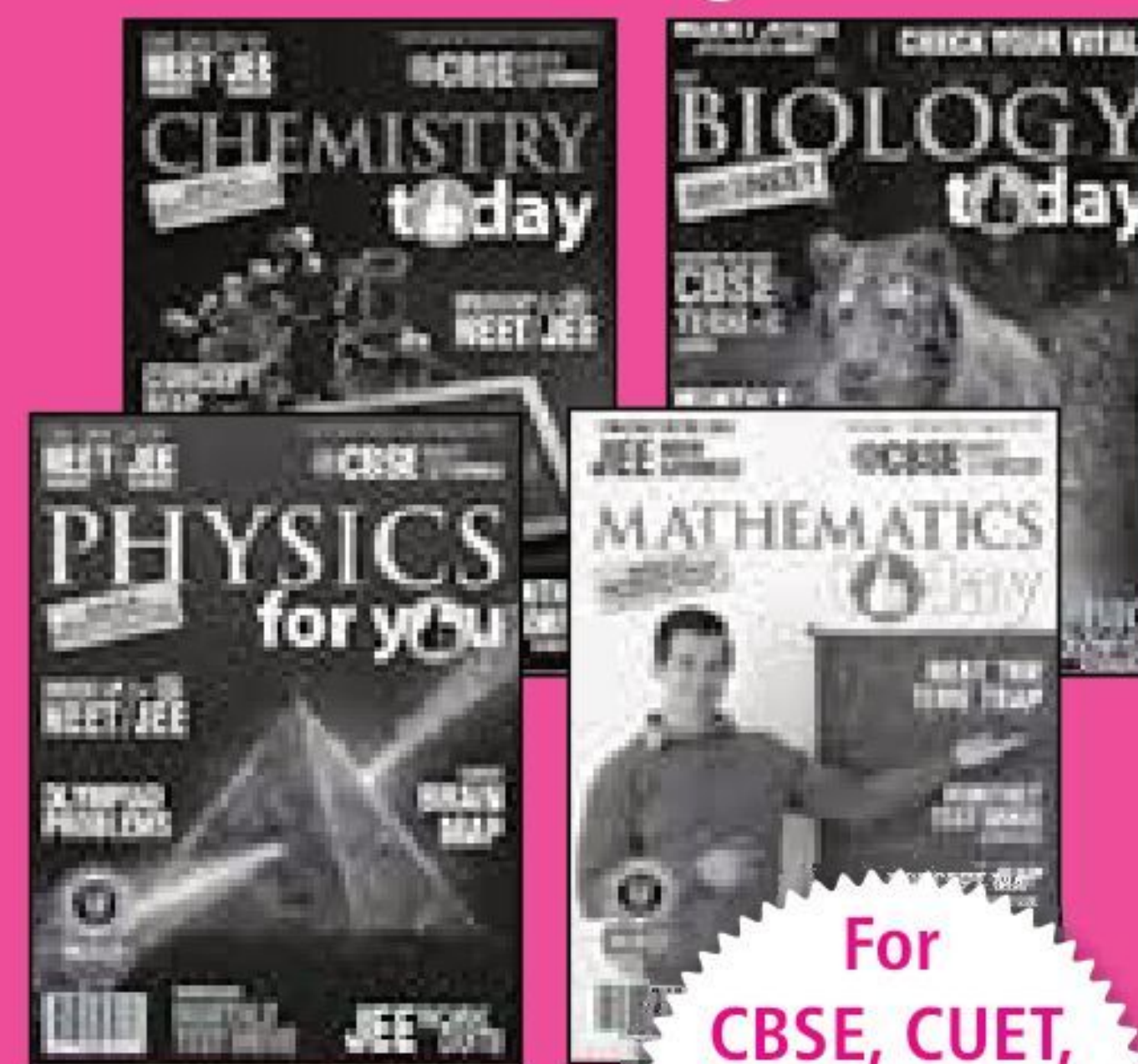


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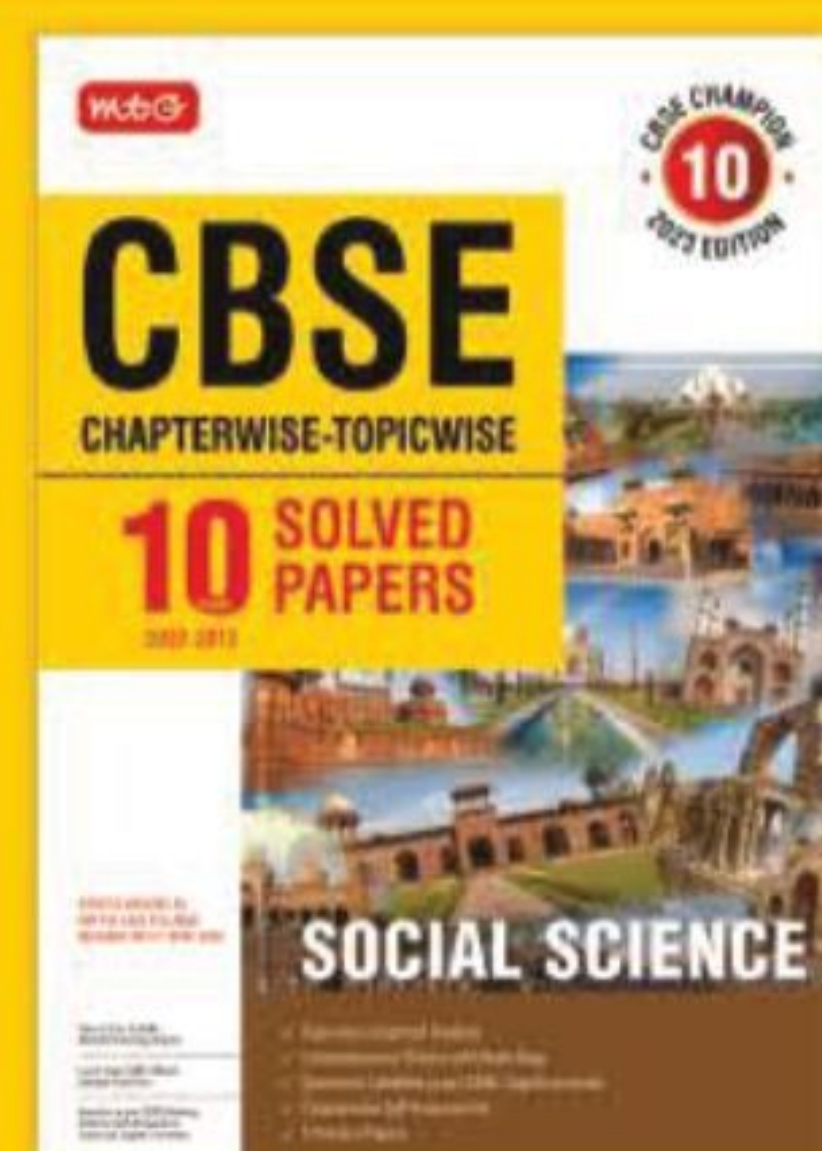
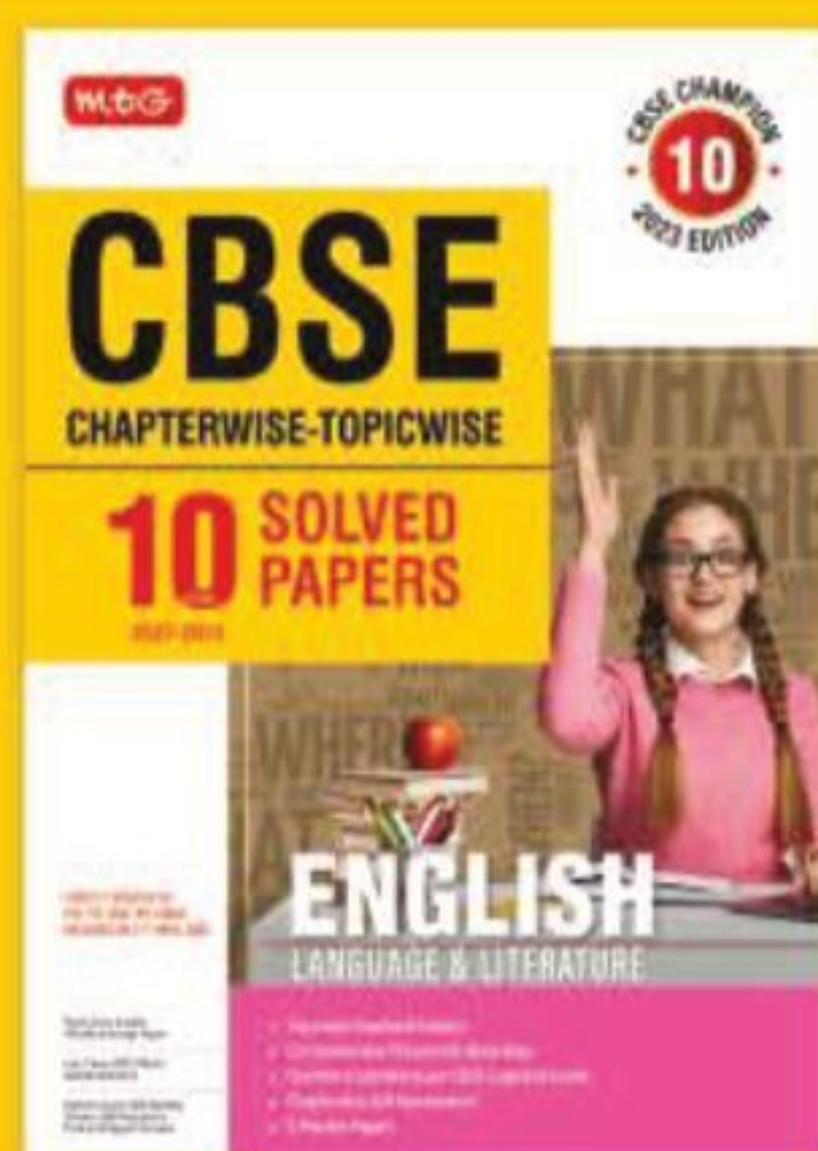
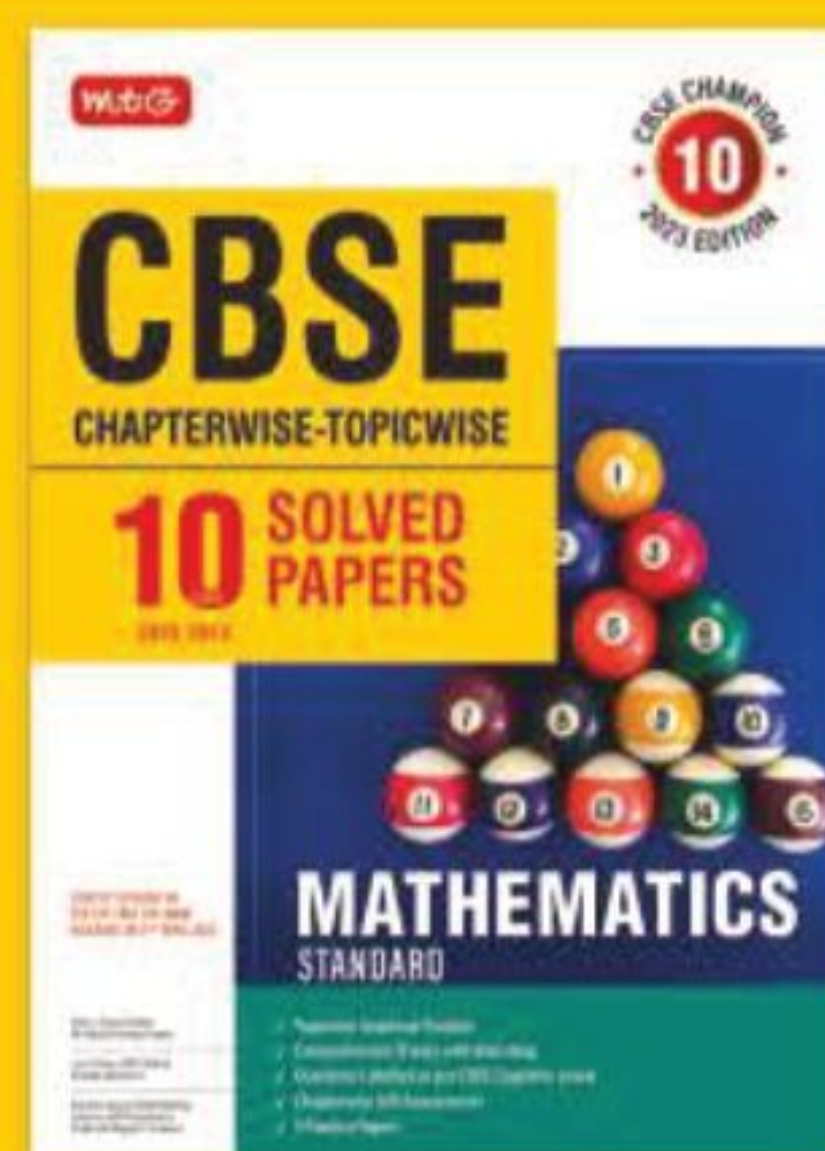
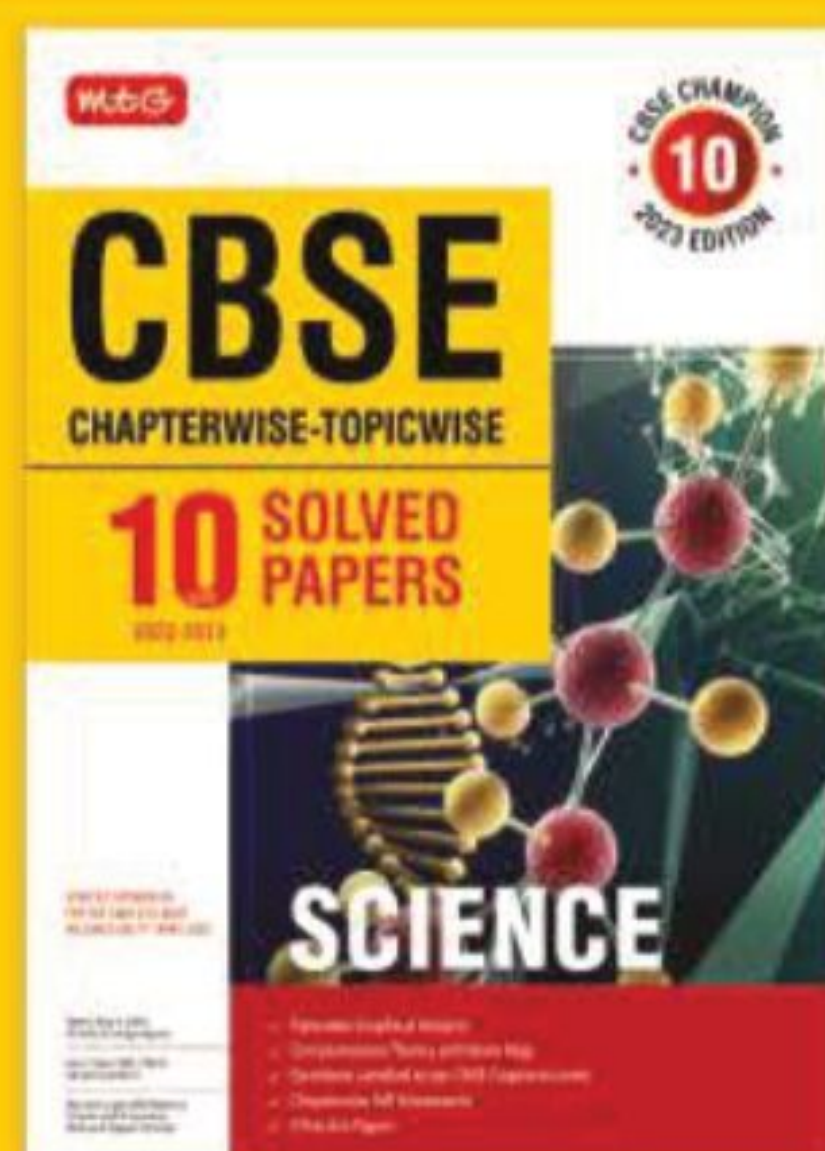
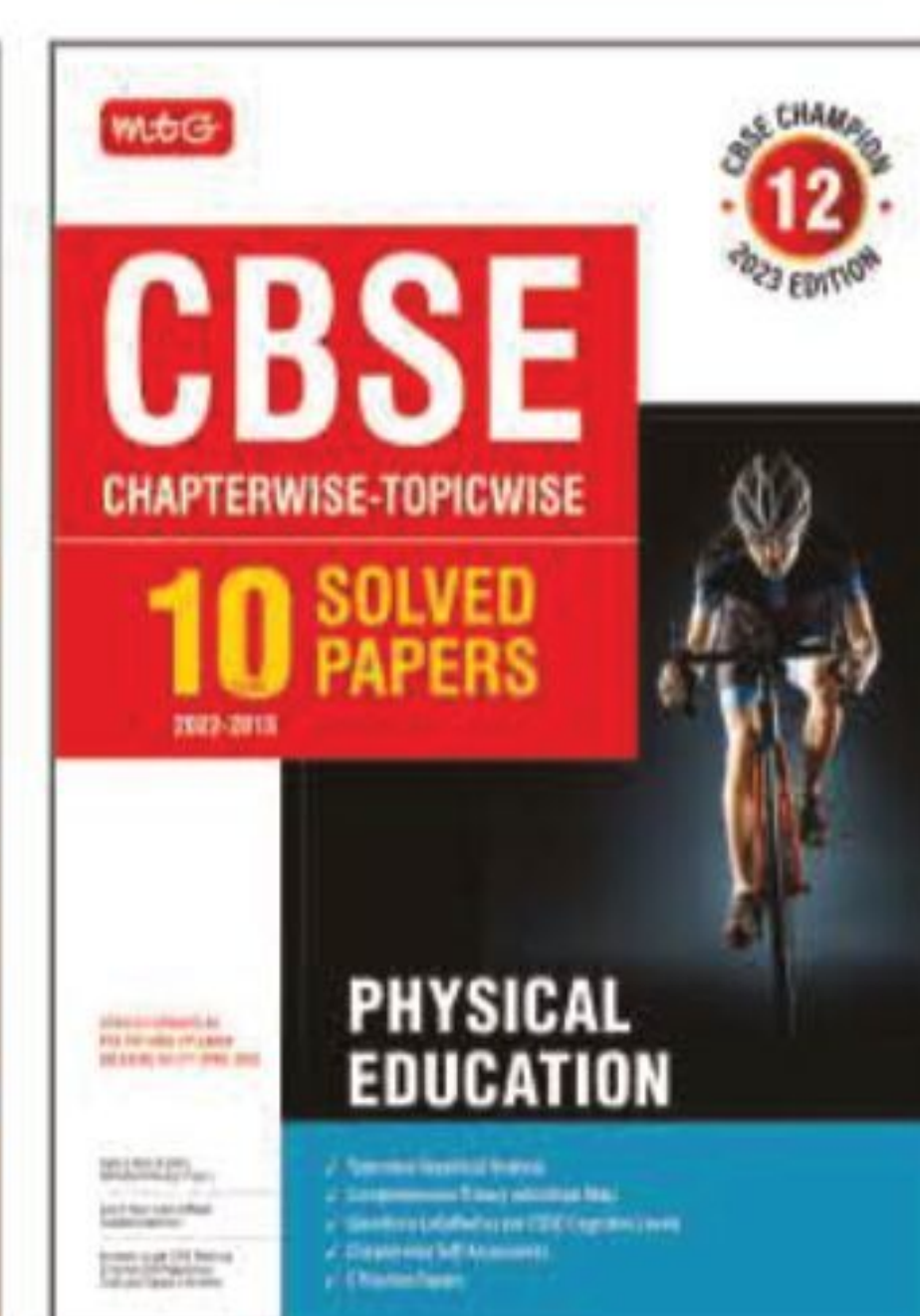
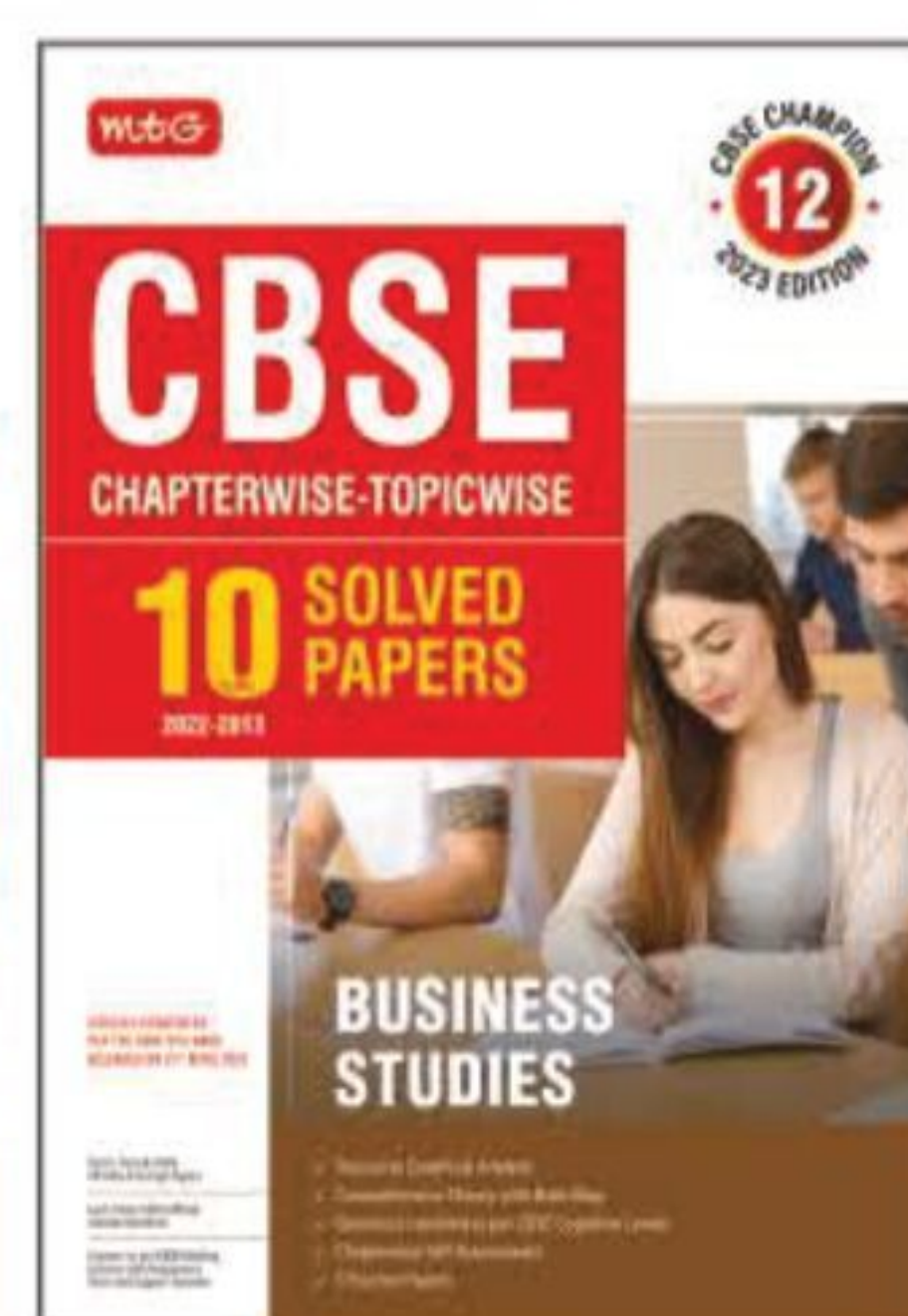
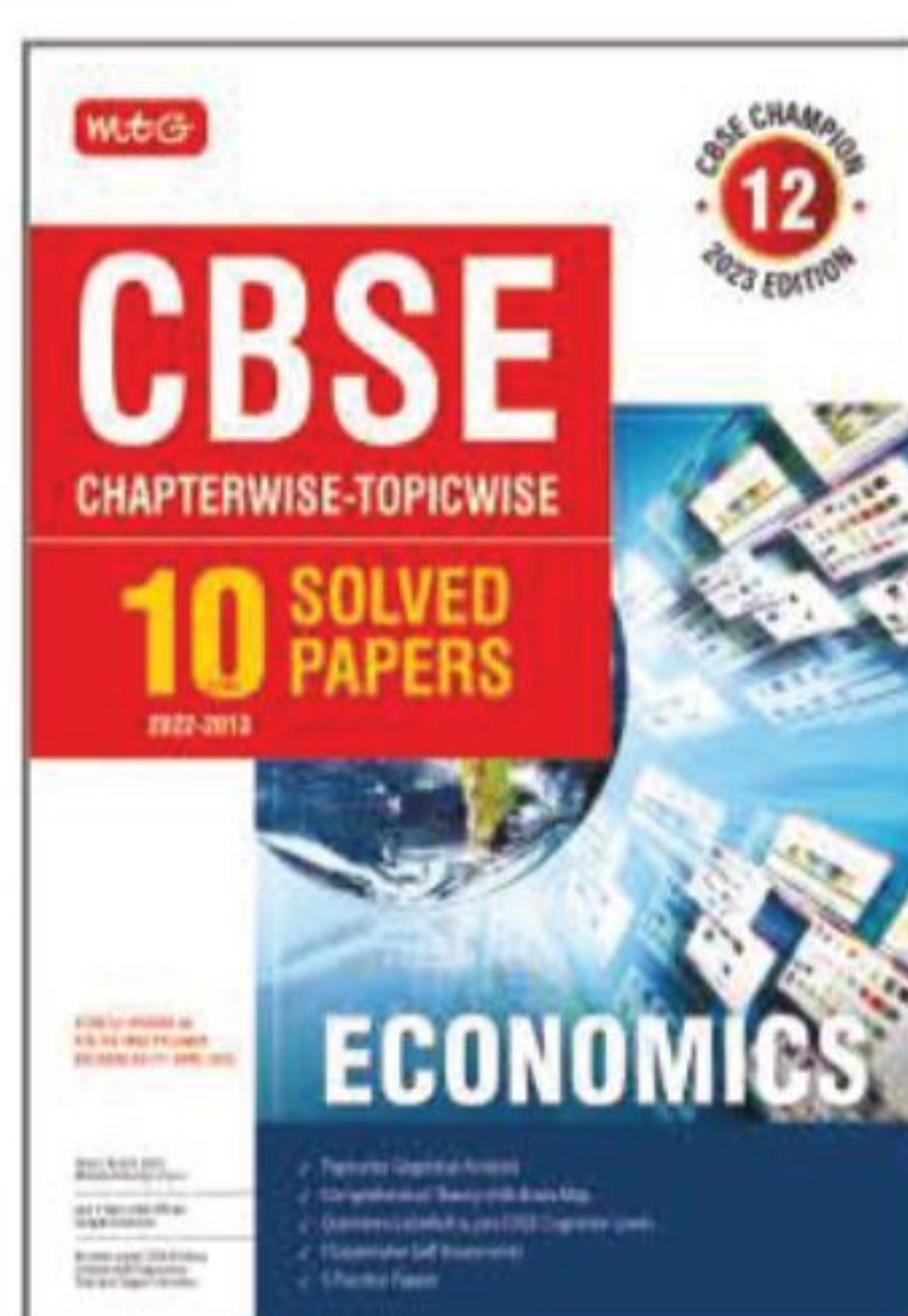
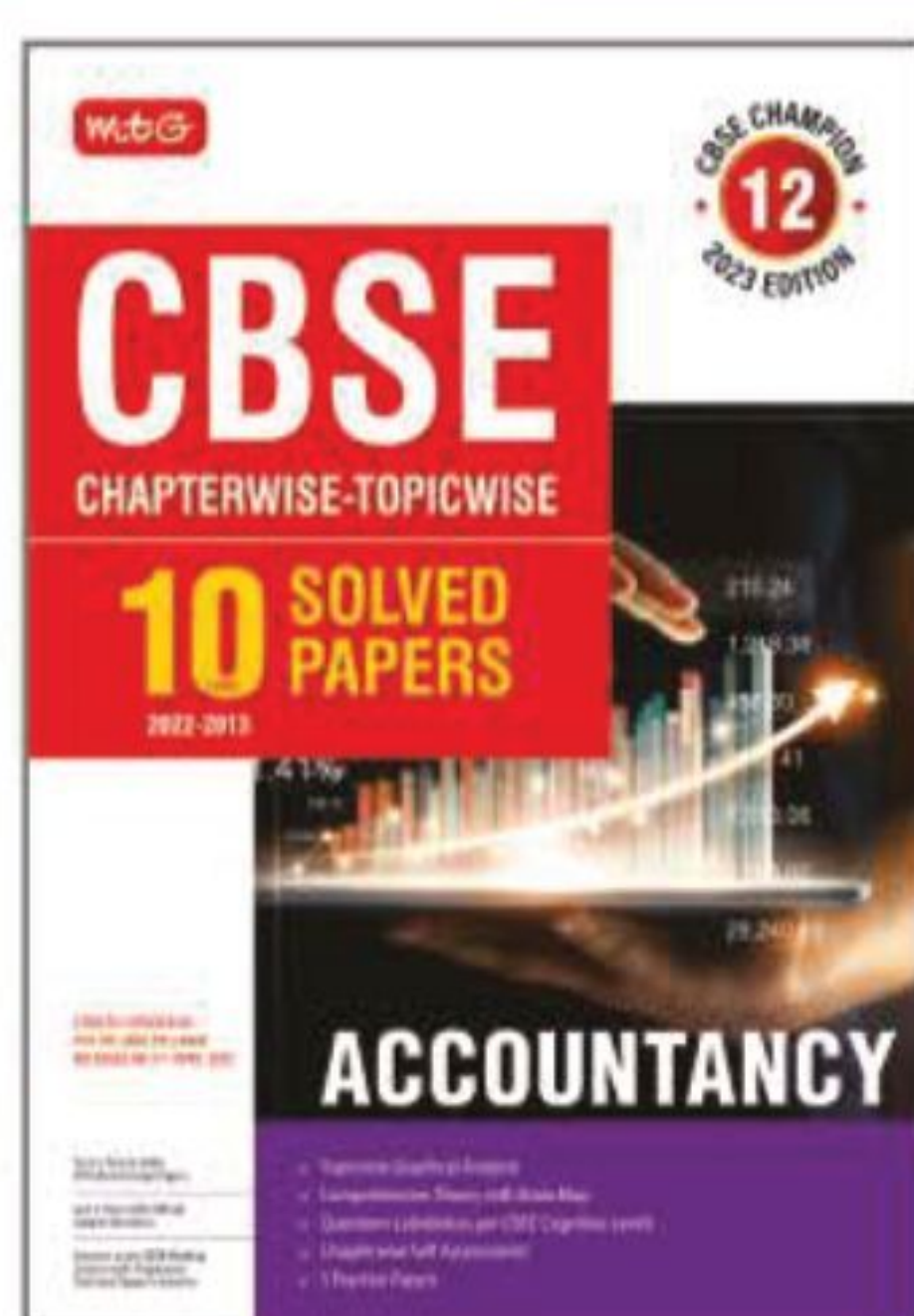
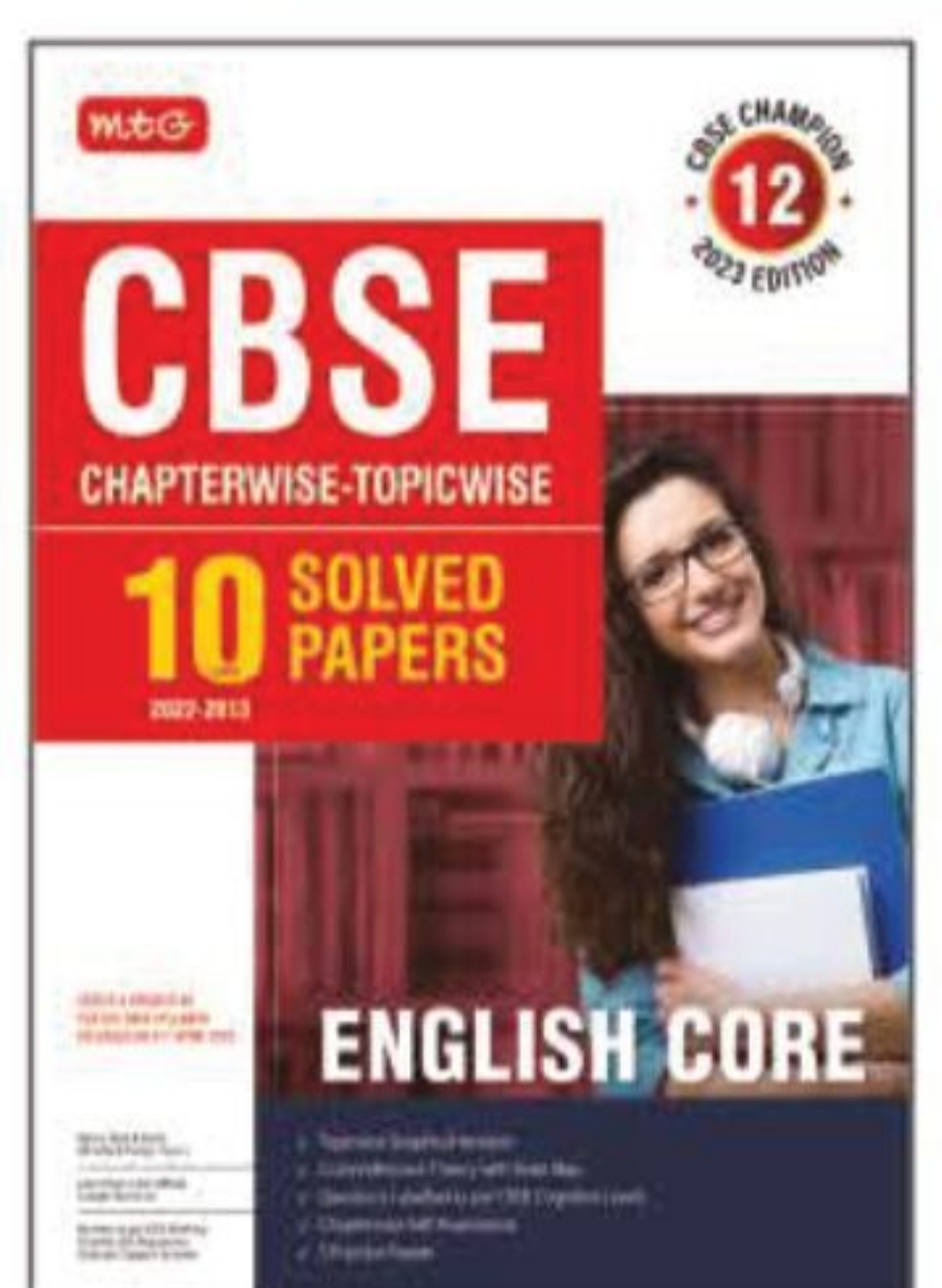
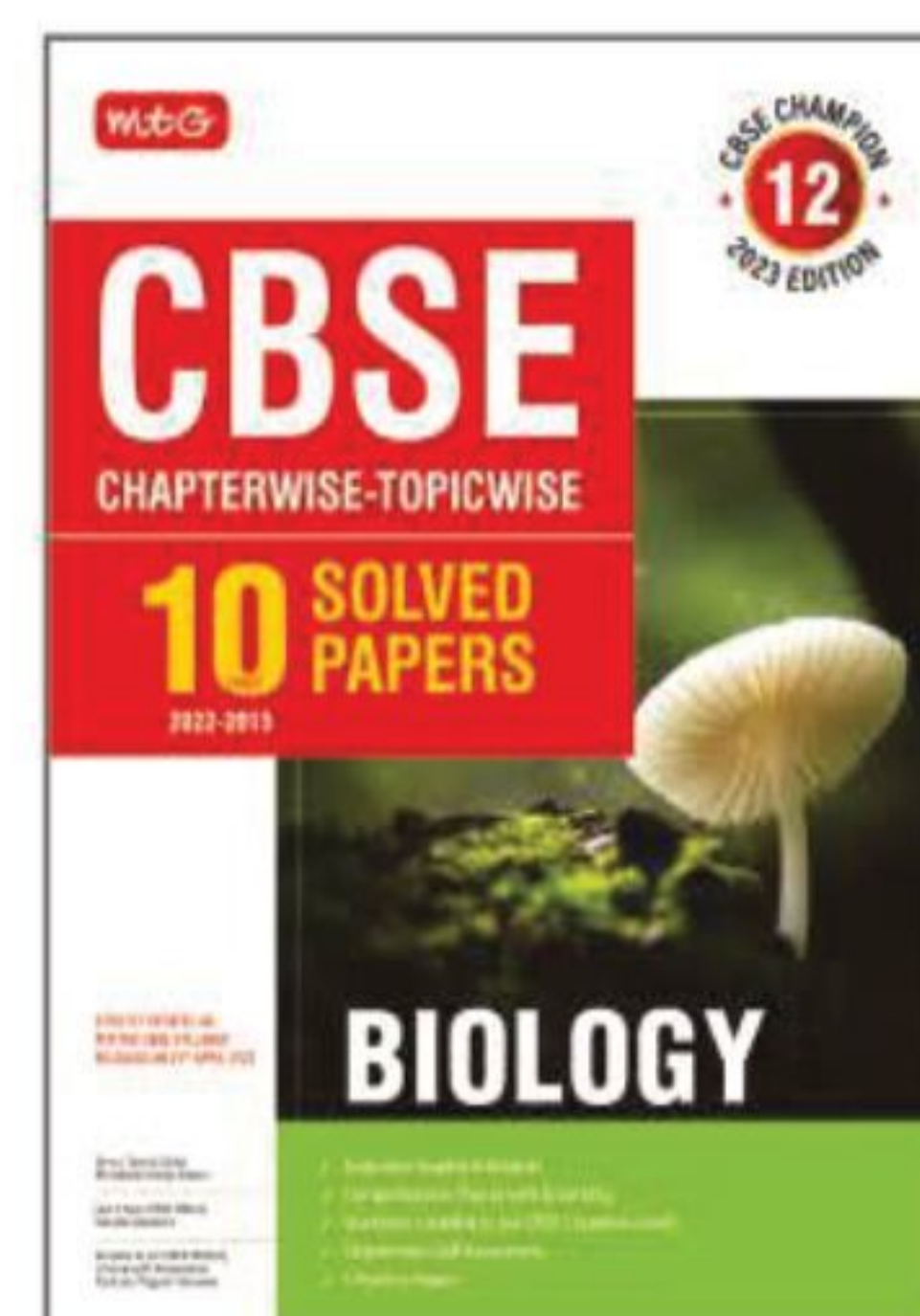
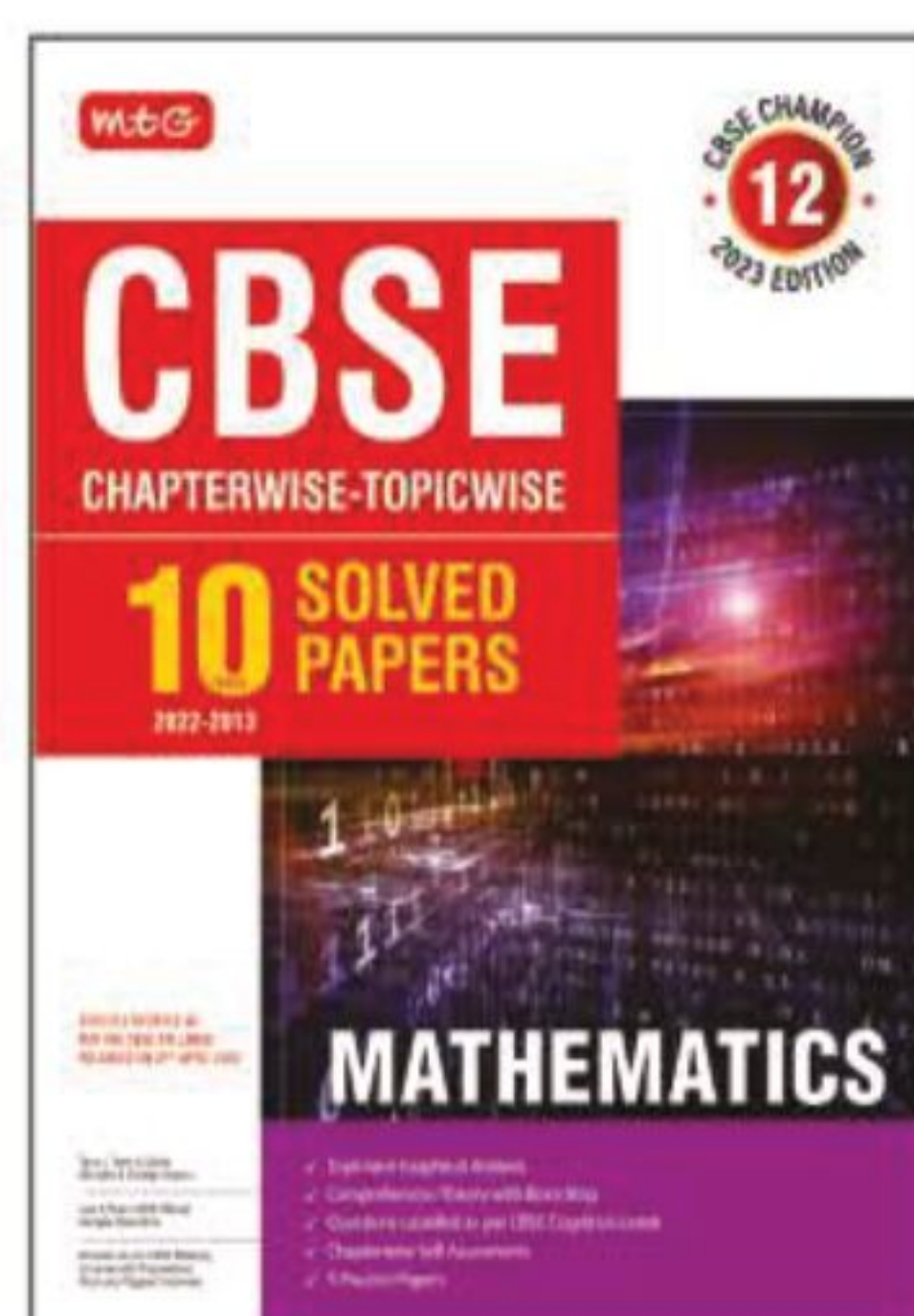
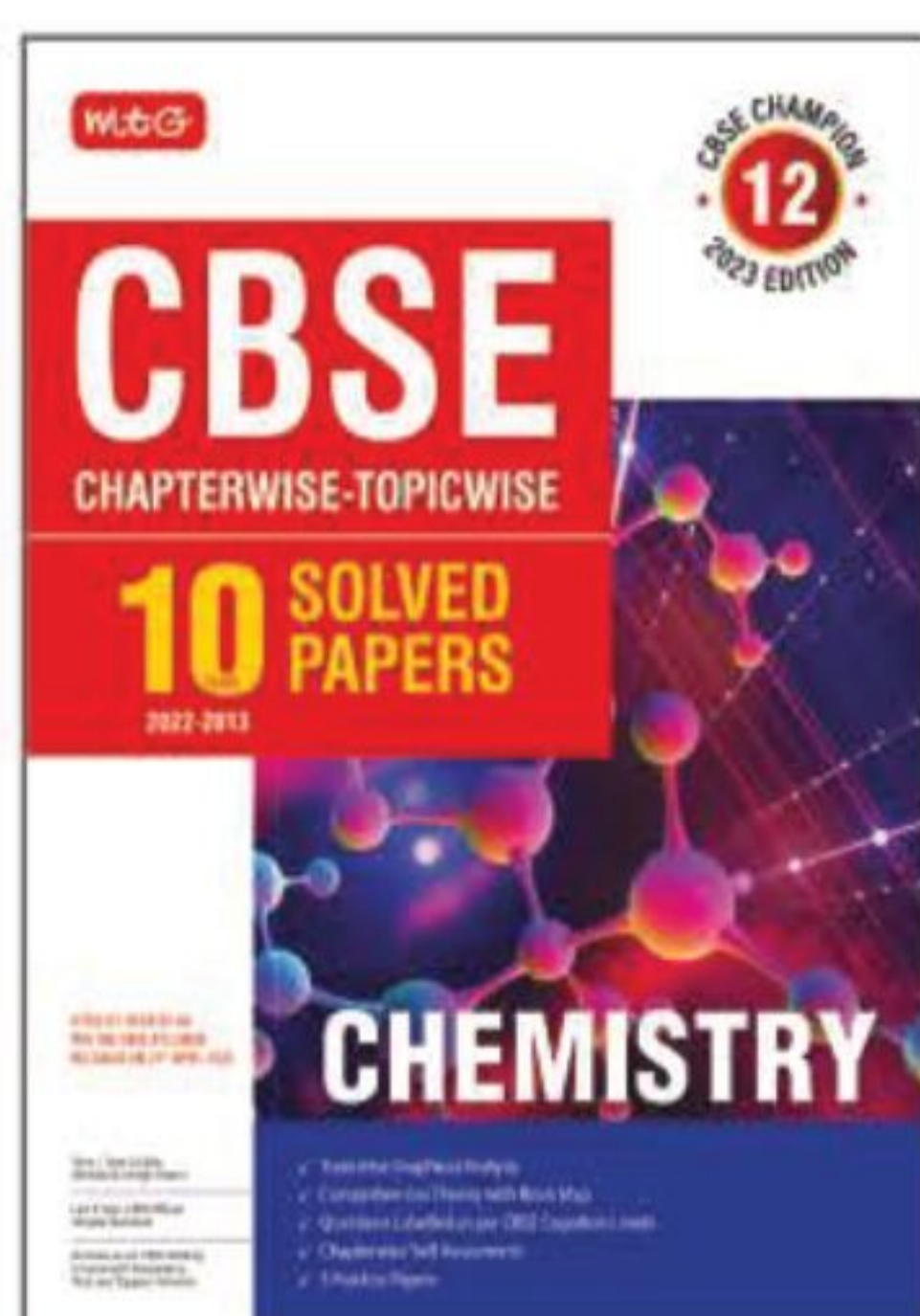
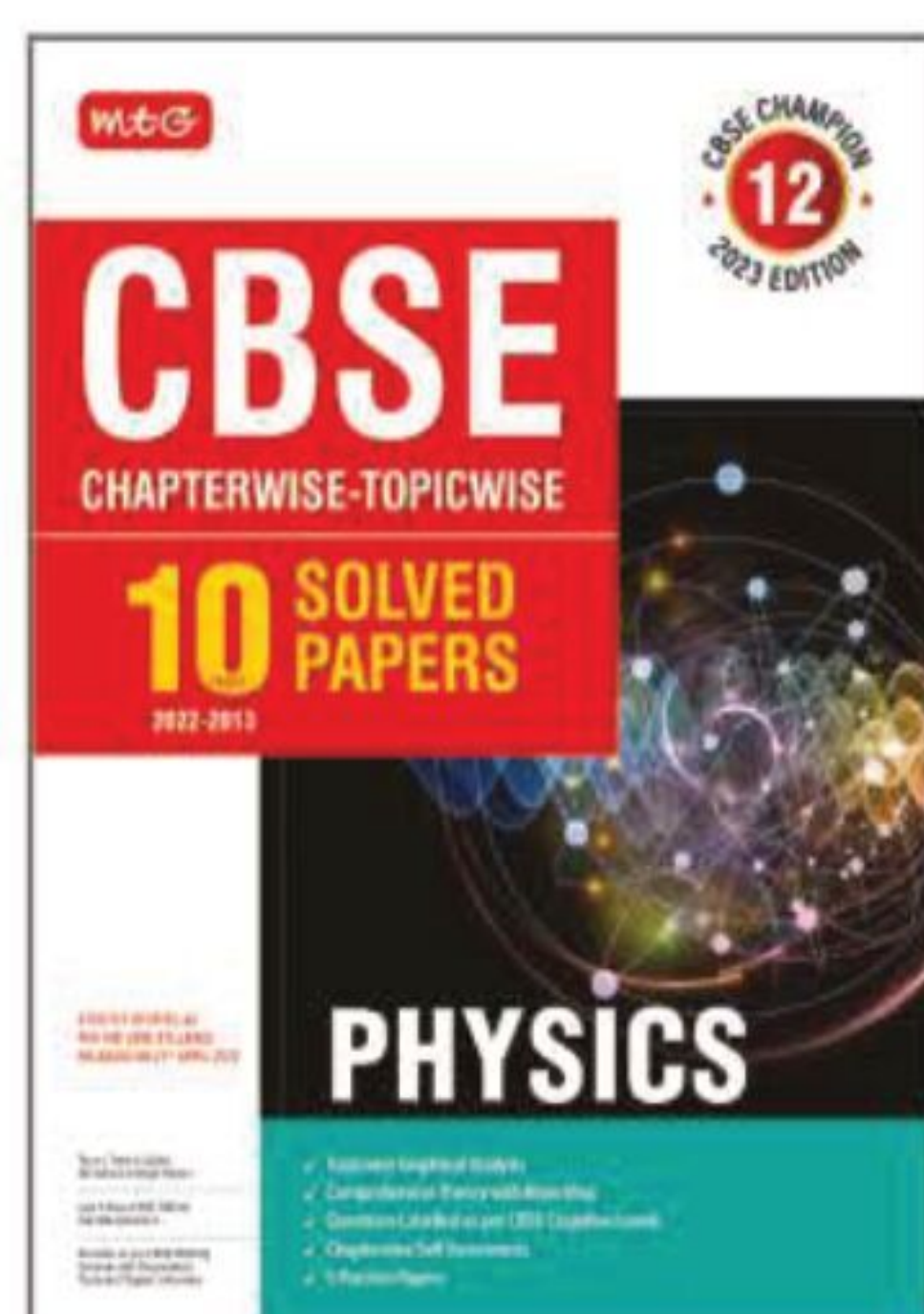
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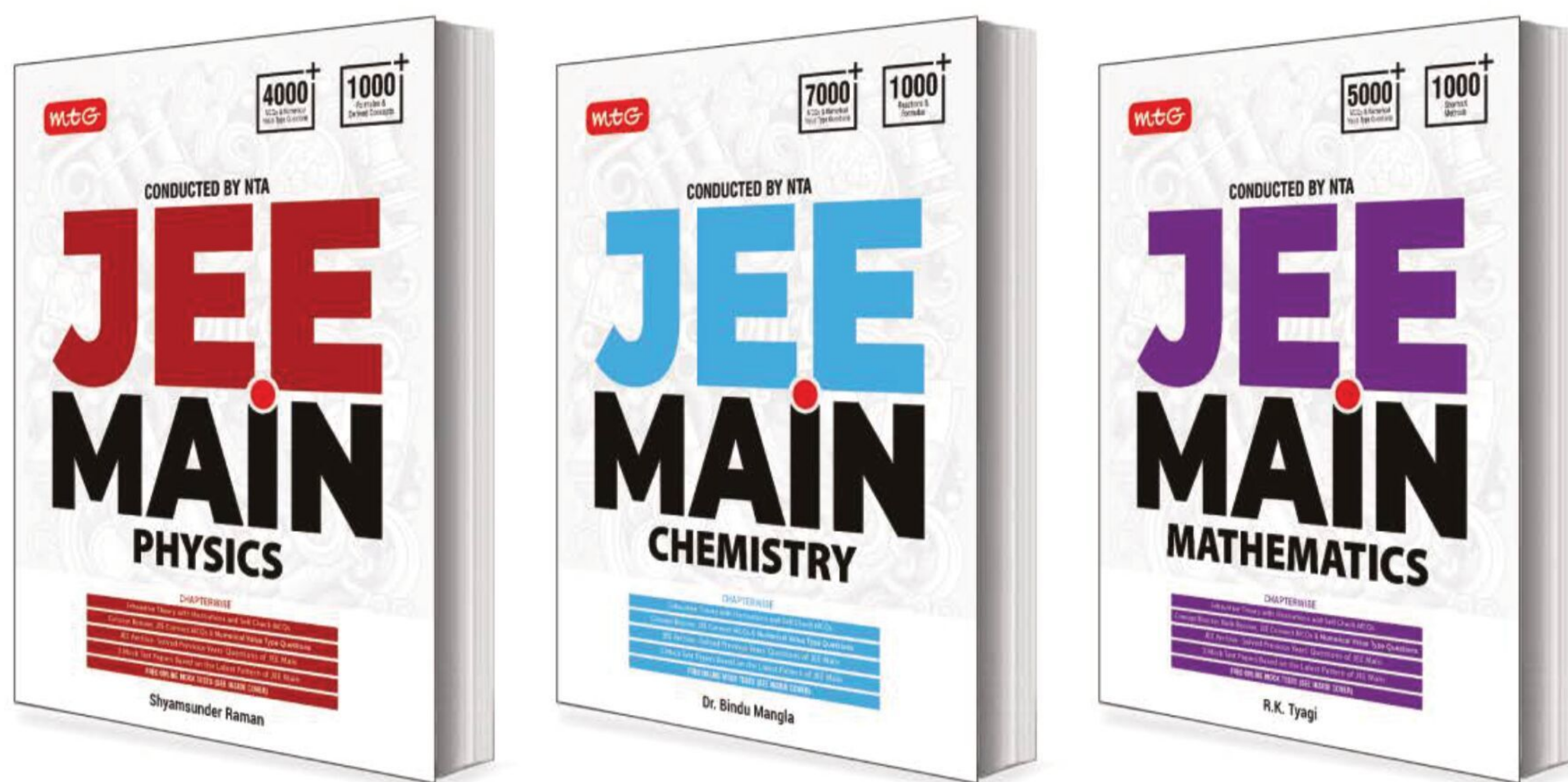
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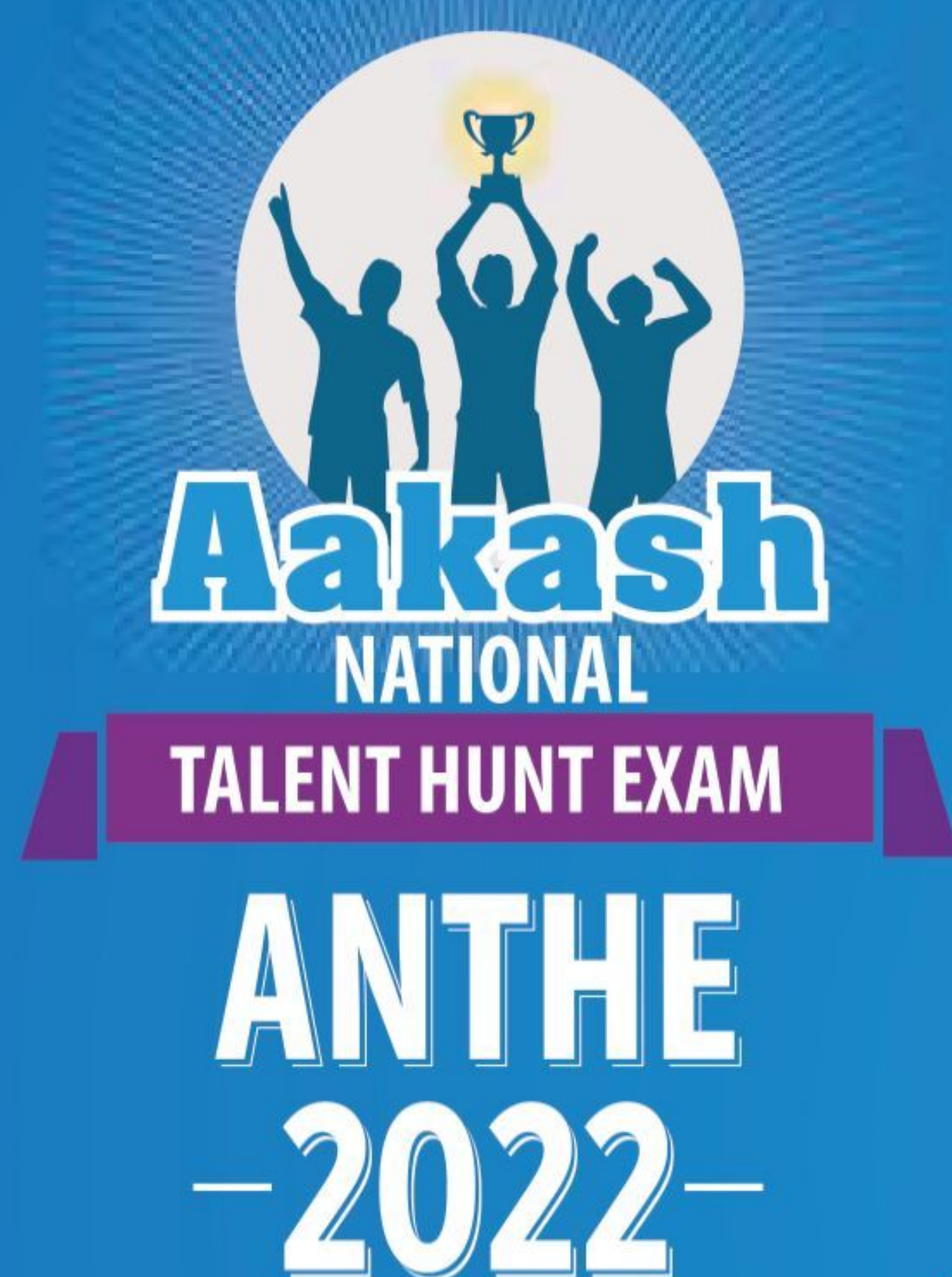
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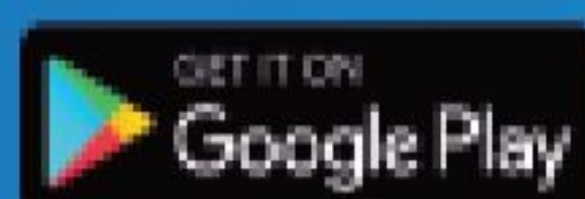
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